4.9 Viscosity: Definition and simple implications

The standard definition of viscosity is illustrated in Fig. (4.6). One wall is moving in z-direction with speed v_z, induced by a force F_z. The second wall is fix. A linear reduction of speed v_z of the viscous liquid is observed implying a momentum current density flowing from left to right.

Experimentally a relation between force F_z and velocity v_z in z-direction is found to be

$$F_x = \eta A \frac{v_x}{z} \quad , \tag{4.70}$$

where A is the cross section, x the distance between the two walls, and the viscosity η is a linear scaling factor. For small distances z the Eq. (4.70) translates into

$$F_x = \eta A \frac{\partial v_x}{\partial z} \quad , \tag{4.71}$$

giving rise to the precise definition of the viscosity η. The linear reduction of v_x in z-direction can easily be understood from the continuity equation (4.57) which for the above case (and adding sources and drains) reads

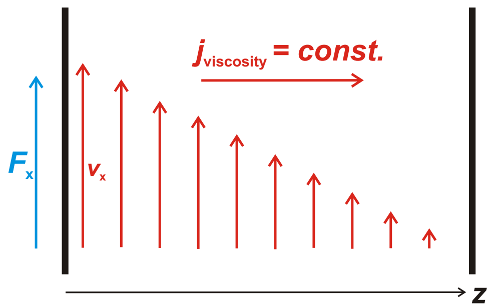

Figure 4.6: Momentum (speed) distribution in z direction between two walls of different speed in x direction.

$$\frac{\partial(\rho v_z)}{\partial t} = -\frac{\partial j_{viscosity}}{\partial z} + \text{sources - drains} \ . \tag{4.72}$$

here $\rho = m/V$ is the mass density.

Temporal changes of momentum are induced by forces; so possible sources and drains in Eq. (4.72) are additional forces. Since no such forces are applied we find for steady state, i.e. $\partial(\rho v_x)/\partial t = 0$,

$$0 = \frac{\partial j_{viscosity}}{\partial z} \quad , \text{i.e.} \quad j_{viscosity} = const. \tag{4.73}$$

So exactly as for diffusion in Eq. (4.56) a linear relation is found for which the continuity equation is the underlying reason.