4.1 Kinetic model of gases

The properties of (ideal) gases will be derived just from the velocity (distribution) of the particles and the collisions with other particles and walls.
The assumptions for this ideal model are

- The gas consists of particles in random motion (no preferred direction).
- The sizes of the particles are negligible (compared to their mean free path).
- Particles interact via brief (10^{-12}s), infrequent and elastic collisions.

The pressure origins from the impact of particles on the container walls, i.e. the total change of linear momentum:

$$p_{\Delta t} = \frac{F}{A} = \frac{m\Delta v}{A\Delta t} = \frac{1}{A}\frac{\Delta P}{\Delta t} = \frac{1}{A\Delta t}\sum_j 2mv_{xj} \quad ; \tag{4.1}$$

the factor of 2 shows up because the wall with an infinitely large mass (compared to the mass of the particles) will just invert the x - component of speed, as a consequence of momentum and energy conservation. Here we take into account the distribution of speed, i.e. the x components of the speed v_{xi} of each particle i.
For discrete speeds the averaging procedure means

$$\overline{v_x} = \frac{1}{N}\left(N_1 v_{x1} + N_2 v_{x2} + ...\right) = \sum_j \frac{N_j}{N} v_{xj} = \sum_j p_j v_{xj} \quad ; \tag{4.2}$$

here j counts the discrete values of v_x and p_j is the probability to obtain v_{xj} in a single measurement.
For a continuous distribution of speed a continuous function for the probability is needed:

$$\overline{v_x} = \sum_{\Delta v_x} v_x p(v_x) = \sum_{\Delta v_x} v_x f(v_x)\Delta v_x = \int_{-\infty}^{+\infty} v_x f(v_x) dv_x \quad . \tag{4.3}$$

Here $f(v_x)dv_x$ is the probability that the x-component of the velocity is within $[v_x, v_x + dv_x]$.
Correspondingly in 3D the probability density or distribution of speeds is

$$f(v_x, v_y, v_z)dv_x dv_y dv_z = \frac{dN(v_x, v_y, v_z)}{N} \quad . \tag{4.4}$$

The fraction of particles with speeds within $[v_x, v_x + dv_x]$, $[v_y, v_y + dv_y]$, $[v_z, v_z + dv_z]$ is

$$f(v_x, v_y, v_z)dv_x dv_y dv_z \quad . \tag{4.5}$$

Assuming a homogenous (number) density of particles n^* we find for the number dN inside a volume fraction dV

$$dN = N\frac{dV}{V} = n^* dV \quad , \tag{4.6}$$

and thus the number of particles with speeds within $[v_x, v_x + dv_x]$, $[v_y, v_y + dv_y]$, $[v_z, v_z + dv_z]$ to be

$$n^* f(v_x, v_y, v_z)dv_x dv_y dv_z \quad . \tag{4.7}$$

The distribution of speed f has the following properties:

- f is normalized
$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(v_x, v_y, v_z)dv_x dv_y dv_z = 1 \quad . \tag{4.8}$$
- f is symmetric, e.g. in x
$$f(v_x, v_y, v_z) = f(-v_x, v_y, v_z) \quad \Rightarrow \quad \int_{-\infty}^{+\infty} f(v_x, v_y, v_z)dv_x = 2\int_0^{+\infty} f(v_x, v_y, v_z)dv_x \quad . \tag{4.9}$$
- f is used to calculate average values, e.g.
$$\overline{E_{kin}} = \frac{m}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(v_x^2 + v_y^2 + v_z^2\right) f(v_x, v_y, v_z)dv_x dv_y dv_z = \frac{m}{2}\overline{v^2} \quad . \tag{4.10}$$
- f is spherically symmetric
$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}\overline{v^2} \quad , \text{with:} \quad v^2 = v_x^2 + v_y^2 + v_z^2 \quad . \tag{4.11}$$