3.26 Maxwell relations

The fundamental equations represent exact differentials, e.g.

$$dG = V dp - SdT \quad \text{, i.e.} \quad \left(\frac{\partial^2 G}{\partial p \partial T}\right) = \left(\frac{\partial^2 G}{\partial T \partial p}\right) \quad \text{; thus} \quad -\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p \tag{3.62}$$

Generally

$$df(x,y) = gdx + hdy \quad \text{gives} \quad \left(\frac{\partial g}{\partial y}\right)_x = \left(\frac{\partial h}{\partial x}\right)_y \tag{3.63}$$

So from the other fundamental equations we get further Maxwell relations:

$$\begin{aligned} \left(\frac{\partial S}{\partial V}\right)_T &= \left(\frac{\partial p}{\partial T}\right)_V \\ \left(\frac{\partial T}{\partial V}\right)_S &= -\left(\frac{\partial p}{\partial S}\right)_V \\ \left(\frac{\partial T}{\partial p}\right)_S &= \left(\frac{\partial V}{\partial S}\right)_p \\ \left(\frac{\partial V}{\partial T}\right)_p &= -\left(\frac{\partial S}{\partial p}\right)_T \end{aligned} \tag{3.64}$$

These Maxwell relations allow for drastic simplification and replacements of difficult-to-measure relations by easy-to-measure relations and thus contribute strongly to the powerful toolbox of thermodynamics as we will see in some subsequent examples.