3.25 Fundamental equations and exact differentials

We now relate Eq. (3.60) to the partial derivatives.

$$
\begin{aligned}
dU =&\ TdS - pdV = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV \quad \Rightarrow \quad \left(\frac{\partial U}{\partial S}\right)_V = T \qquad \left(\frac{\partial U}{\partial V}\right)_S = -p \\
dF =&\ -SdT - pdV = \left(\frac{\partial F}{\partial T}\right)_V dT + \left(\frac{\partial F}{\partial V}\right)_T dV \quad \Rightarrow \quad \left(\frac{\partial F}{\partial T}\right)_V = -S \qquad \left(\frac{\partial F}{\partial V}\right)_T = -p \\
dH =&\ TdS + Vdp = \left(\frac{\partial H}{\partial S}\right)_p dS + \left(\frac{\partial H}{\partial p}\right)_S dp \quad \Rightarrow \quad \left(\frac{\partial H}{\partial S}\right)_p = T \qquad \left(\frac{\partial H}{\partial p}\right)_S = V \\
dG =&\ -SdT + Vdp = \left(\frac{\partial G}{\partial T}\right)_p dT + \left(\frac{\partial G}{\partial p}\right)_T dp \quad \Rightarrow \quad \left(\frac{\partial G}{\partial T}\right)_p = -S \qquad \left(\frac{\partial G}{\partial p}\right)_T = V
\end{aligned}
\tag{3.61}
$$

These results clarify which exact differential with which restrictions have to be used/combined to get proper results related to the thermodynamic potentials.