3.22 Calculation of the free energy of an ideal gas

We can calculate the free energy (Helmholtz energy) starting with the state function and the inner energy of an ideal gas

$$pV = NkT \qquad , \tag{3.50}$$

$$U = 3/2NkT \qquad . \tag{3.51}$$

(Note: In this notation U is not a potential, since T is not a coordinate of U!). Using Eq. (3.50) we get

$$p = -\frac{\partial F}{\partial V} = \frac{NkT}{V} \qquad , \tag{3.52}$$

i.e.

$$F(V, N, T) = -NkT(\ln(V) + K(N, T)) \qquad . \tag{3.53}$$

The function $K(N, T)$ must still be calculated. Combining Eq. (3.51),

$$U = F + TS, \text{ and } S = -dF/dT \tag{3.54}$$

we find

$$\begin{aligned} \frac{3}{2}NkT \quad = \quad U &= -NkT\left(\ln(V) + K(N, Z)\right) - T\left[-Nk\left(\ln(V) + K(N, T)\right) - NkT\frac{\partial K(T,N)}{\partial T}\right] \\ &= NkTT\frac{\partial K(T,N)}{\partial T} \qquad . \end{aligned} \tag{3.55}$$

Consequently

$$K(T, N) = 3/2 \ln(T) + K'(N) \qquad , \tag{3.56}$$

leading to

$$F(V, N, T) = -NkT\left[\ln(V) + 3/2 \ln(T) + K'(N)\right] \qquad . \tag{3.57}$$

Successively integrating the state functions of a system allows to calculate the thermodynamic potential.
This procedure is necessary because in contrast to an electrical potential there is no way of measuring a thermodynamic potential directly. We therefore have to measure all "forces" in each state, thus determining the state functions which allow us to calculate the potential.