3.20 The Legendre transformation in 1D

We investigate the function $y(x)$ and $z := dy/dx$.
The "total differential" is

$$dy = zdx \qquad . \tag{3.40}$$

Calculating

$$F(z) = y(x(z)) - zx(z) \tag{3.41}$$

we find its derivation

$$dF/dz = dy/dx(z)dx/dz(z) - x(z) - zdx/dz(z) = -x(z) \qquad , \tag{3.42}$$

the "total differential" is

$$dF = -xdz \qquad . \tag{3.43}$$

The transformation in Eq. (3.41) is called **Legendre transformation**. A coordinate is replaced by its force. The main advantage of this transformation is the inverse transformation (Legendre transformation of $F(z)$). We find:

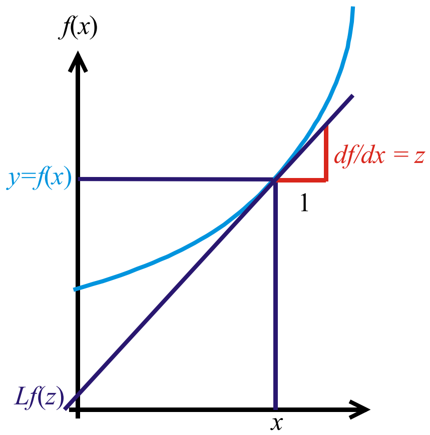
$$G(x) = F(z(x)) - (-x)z(x) = y(x(z(x))) - z(x)x(z(x)) + xz(x) = y(x) \qquad , \tag{3.44}$$

which is the original function **without any loss of information**.
Neglecting the additional minus sign of the inverse transformation, the pair

coordinate ⇔ force

is absolutely symmetric; e.g. depending on the potential $-p$ is a force, respectively p is a coordinate.

- Graphical representation of the Legendre transformation ⟶
- Description of the curve by the "wrapping tangents"
- For each x only one slope z must exist in order to get a well defined inverse function
- What would happen if for a given pressure two possible volumes would exist? (not possible!!, not stable!!)
- Thermodynamic functions are always convex and therefore stable



- The Legendre transformation allows to transform within a potential from the intensive to the extensive parameter (and vice versa) without loss of information. This calculated potential automatically describes the corresponding thermodynamic contact correctly.
- The Legendre transformation can be applied to any coordinate independently.
- All contacts can thus be described when knowing one thermodynamic potential of the system for just one thermodynamic contact.