3.19 The transformation of thermodynamic potentials

Why not just replace a coordinate by its adjacent force?
We will have a closer look at a 1D example of a function $y(x)$.
Let

$$z := dy/dx \tag{3.34}$$

and

$$x = x(z) \quad . \tag{3.35}$$

We find

$$y = y(x(z)) = f(z) \quad , \tag{3.36}$$

consequently

$$y = f(dy/dx) \quad , \tag{3.37}$$

respectively

$$dy/dx = f^{-1}(y) \quad . \tag{3.38}$$

This differential equation has the solution $y(x)$; but there exist more solutions

$$y = y(x + const.) \quad . \tag{3.39}$$

Simply substituting a coordinate x by the adjacent force z leads to a **loss of information**, since the original function is known only up to a constant. For a function with several parameters this is even more critical since the "constant" may by a function of all the remaining parameters.