5.4 Current flow through a non degenerated semiconductor

We investigate the charge flow in an electrical field for a non degenerated semiconductor. Combining the equations (5.18) and (5.21) we get

$$\vec{j} = -\frac{q}{4\pi^3} \int_{V_k} \tau(\vec{k})\vec{v}(\vec{k}) \frac{\partial f_0}{\partial E} \left\langle \left(q\vec{E} - \vec{\nabla}_r \mu - (E-\mu)\vec{\nabla}_r \ln(T) \right), \vec{v} \right\rangle d^3k \quad . \tag{5.24}$$

We will neglect gradients in the temperature, so $(E-\mu)\vec{\nabla}_r \ln(T)$ vanishes. Since the semiconductor is not degenerated we can simplify the Fermi statistics by the Boltzmann statistics, i.e.

$$\frac{\partial f_0}{\partial E} = -\frac{f_0}{kT} \quad . \tag{5.25}$$

We take the band energies of the free electron gas

$$E = E_0 + \frac{m^*}{2} \left| \vec{v}(\vec{k}) \right|^2 \text{, so} \quad v_i = \frac{\hbar}{m^*} k_i \quad . \tag{5.26}$$

In addition we assume τ to be independent of $\vec{k}$. Summing up all approximations we find for the particle current

$$\vec{j} = -\frac{q\tau\hbar^2}{4\pi^3 kT(m^*)^2} \left[e\vec{E} - \vec{\nabla}\mu \right] \tilde{M} \quad , \tag{5.27}$$

and $\tilde{M}$ is a matrix with the components

$$\tilde{M}_{ij} = \int_{V_k} k_i k_j f_0(E(k)) d^3k = \int_k dk k^4 f_0(E(k)) \int_{S_k} d\Omega \frac{k_i k_j}{k^2} \quad . \tag{5.28}$$

Using $k^2 = k_x^2 + k_y^2 + k_z^2$ and integrating over the surface of a sphere we get

$$\int_{S_k} d\Omega \frac{k_i k_j}{k^2} = \frac{4\pi}{3} \delta_{ij} \quad . \tag{5.29}$$

Using

$$dE = \frac{\hbar^2}{m^*} k dk \quad , \tag{5.30}$$

the current density is written as

$$\vec{j} = -\frac{q\tau}{3\pi^2 kT m^*} \left[q\vec{E} - \vec{\nabla}\mu \right] \left(\frac{2m^*}{\hbar^2} \right) \int_{E_0}^{\infty} dE (E-E_0)^{\frac{3}{2}} \exp\left(-\frac{E-\mu}{kT} \right) \quad . \tag{5.31}$$

After partial integration we get

$$\vec{j} = -\frac{q\tau}{3\pi^2 kT m^*} \left[q\vec{E} - \vec{\nabla}\mu \right] \left(\frac{2m^*}{\hbar^2} \right) \frac{-3kT}{2} \int_{E_0}^{\infty} dE (E-E_0)^{\frac{1}{2}} \exp\left(-\frac{E-\mu}{kT} \right) \quad . \tag{5.32}$$

Taking into account the density of state of free electrons

$$D(E) = \frac{1}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{\frac{3}{2}} (E-E_0)^{\frac{1}{2}} \quad , \tag{5.33}$$

we finally get

$$\vec{j} = \frac{q\tau}{m^*} \left[q\vec{E} - \vec{\nabla}\mu \right] \int_{E_0}^{\infty} dE D(E) f_0(E) = \frac{q\tau n_e}{m^*} \left[q\vec{E} - \vec{\nabla}\mu \right] \quad . \tag{5.34}$$