5.2 The relaxation time approximation in the Boltzmann equation

At time $t = 0$ we will switch of all external forces of the system. By scattering the system reaches the thermodynamic equilibrium state again, which will be described in linear approximation by

$$\left(\frac{\partial f}{\partial t}\right) = \left(\frac{\partial f}{\partial t}\right)_{scat} = -\frac{f(\vec{r},\vec{k},t) - f_0(\vec{r},\vec{k})}{\tau(\vec{k})} \tag{5.10}$$

$f_0(\vec{r},\vec{k})$ is the equilibrium distribution function.
The relaxation time $\tau(\vec{k})$ describes how fast the system reaches thermodynamic equilibrium again.
The solution of the relaxation process is:

$$f(\vec{r},\vec{k},t) - f_0(\vec{r},\vec{k}) = \left[f(\vec{r},\vec{k},0) - f_0(\vec{r},\vec{k})\right] e^{-\frac{t}{\tau(\vec{k})}} \tag{5.11}$$

The essence for the following calculation is that this relaxation time does not depend on the external forces (This is a very strong assumption; it does not hold e.g. in the space charge region or for "injection level spectroscopy"). For steady state $\left(\frac{\partial f}{\partial t}\right) = 0$ we get

$$-\left(\frac{\partial f}{\partial t}\right)_{field} = \left\langle \vec{\nabla}_r f, \vec{v}\right\rangle + \frac{1}{\hbar}\left\langle \vec{\nabla}_k f, \vec{F}_a\right\rangle = -\frac{f(\vec{r},\vec{k}) - f_0(\vec{r},\vec{k})}{\tau(\vec{k})} \tag{5.12}$$

This is the fundamental equation for the description of stationary processes in relaxation time approximation.
For small perturbations we evaluate in a series:

$$f(\vec{r},\vec{k}) = f_0(\vec{r},\vec{k}) + f^{(1)}(\vec{r},\vec{k}) + f^{(2)}(\vec{r},\vec{k}) + ... \tag{5.13}$$

and consider only the linear terms leading to

$$\left\langle \vec{v}, \vec{\nabla}_r f_0(\vec{r},\vec{k}) + \vec{\nabla}_r f^{(1)}(\vec{r},\vec{k})\right\rangle + \frac{1}{\hbar}\left\langle \vec{F}_a, \vec{\nabla}_k f_0(\vec{r},\vec{k}) + \vec{\nabla}_k f^{(1)}(\vec{r},\vec{k})\right\rangle = -\frac{f^{(1)}(\vec{r},\vec{k})}{\tau(\vec{k})} \tag{5.14}$$

Since the gradients $\vec{\nabla}_r$ and $\vec{\nabla}_k$ depend already linearly on the perturbation the derivations of $f^{(1)}$ are of second order $f^{(2)}$ and are therefor neglected. We find:

$$\vec{\nabla}_r f_0(\vec{r},\vec{k}) = \vec{\nabla}_r \left(\frac{1}{1 + e^{\frac{E(\vec{k})-\mu(\vec{r})}{kT(\vec{r})}}}\right) = -\frac{\partial f_0}{\partial E}\left(\vec{\nabla}_r \mu + (E-\mu)\frac{\vec{\nabla}_r T}{T}\right) \tag{5.15}$$

and

$$\vec{\nabla}_k f_0(\vec{r},\vec{k}) = \vec{\nabla}_k \left(\frac{1}{1 + e^{\frac{E(\vec{k})-\mu(\vec{r})}{kT(\vec{r})}}}\right) = \frac{\partial f_0}{\partial E}\vec{\nabla}_k E(\vec{k}) = \frac{\partial f_0}{\partial E}\hbar\vec{v} \tag{5.16}$$

For an electrical field

$$\vec{F} = q\vec{E} \tag{5.17}$$

we finally get

$$-\frac{f^{(1)}(\vec{r},\vec{k})}{\tau(\vec{k})} = \frac{\partial f_0}{\partial E}\left\langle \left(q\vec{E} - \vec{\nabla}_r \mu - (E-\mu)\vec{\nabla}_r \ln(T)\right), \vec{v}\right\rangle \tag{5.18}$$

The three terms on the right hand side describe

- the ohmic law
- particle diffusion
- heat transport phenomena

An overview of the above described an other time consuming processes is discussed in the semiconductor script.