4.6 Guided tour through synergy

The LASER system shows an remarkable discontinuity:
For low pumping power or using a wrong resonator arrangement the system behaves as a conventional light source:

- radiation in all directions
- "usual" spectral intensity distribution

Above the LASER threshold:

- strongly focused light beam
- only very few Eigen frequencies

The System changes from an **unordered** state to an **ordered** state.

Most characteristic for this phenomenon is the sudden transition: small changes in the pumping power or the mirror arrangement may change the state extremely.

There exist many systems in nature (living and non living) showing this tendency of spontaneous rearrangement (self organization) to higher order.

All systems have in common:

1. They are far from thermodynamic equilibrium
2. This non equilibrium state must be sustained by energy supply
3. It is a strongly non linear system

This ordering systems contradict *not* the second axiom, since the energy supply adds entropy to other regions of the system.

LASER, living organisms, neural nets, Wall Street, ... are examples for such systems.

The most simple non linear differential equation of a LASER

n:	number of photons in the resonator
N_1, N_2:	number of electrons in the LASER energy states

We find

$$\left(\frac{dn}{dt}\right)_+ = G(N_2 - N_1)n \tag{4.34}$$

G: amplification factor.
Correspondingly we get

$$\left(\frac{dn}{dt}\right)_- = -\alpha n \qquad , \tag{4.35}$$

which sums up all diffraction and mirror losses.
Using

$$N = N_2 - N_1 \qquad , \tag{4.36}$$

we find

$$\left(\frac{dn}{dt}\right) = \left(\frac{dn}{dt}\right)_+ + \left(\frac{dn}{dt}\right)_- = GNn - \alpha n \qquad . \tag{4.37}$$

Additionally we have

$$\left(\frac{dN}{dt}\right) = C - \lambda N - \tilde{\alpha} n \tag{4.38}$$

with C: pumping mechanism
λ: relaxation processes
$\tilde{\alpha}$: induced emission and absorption

For steady state condition holds

$$\left(\frac{dN}{dt}\right) = 0 \qquad , \tag{4.39}$$

leading to

$$N = \frac{C}{\lambda} - \frac{\tilde{\alpha}}{\lambda} n = N_0 - \gamma n \quad , \tag{4.40}$$

and finally:

$$\frac{dn}{dt} = G(N_0 - \gamma n)n - \alpha n = -\delta n - \epsilon n^2 \qquad , \tag{4.41}$$

with

$$\delta = \alpha - G N_0 \tag{4.42}$$

and

$$\epsilon = G\gamma \tag{4.43}$$

The solution of this differential equation is

$$n(t) = \frac{1}{c e^{\delta t} - \frac{\epsilon}{\delta}} \qquad . \tag{4.44}$$

$n(t)$ shows extremely different behavior for $\delta > 0$ and $\delta < 0$:

$$\begin{array}{lcl} \delta > 0 \;:\; n(t \to \infty) & = & 0 \\ \delta < 0 \;:\; n(t \to \infty) & = & -\frac{\delta}{\epsilon} \end{array} \tag{4.45}$$

δ is called the ordering parameter which determines the LASER condition.

LASER and LIFE

There exist a lot of analogies between a LASER an biological systems:

LASER	**Life**
Inversion	food supply
LASER condition	to be alive
Mode	Species
ΔE	special food (e.g. grass)
homogeneous emission line	only one kind of food
inhomogeneous emission line	broad spectra of food
homogeneous emission means that only one mode will survive	One species dominates when two (or more) species claim exactly the same food (Darwinism)
spatial hole burning	e.g. different migration habits allow to used different areas at different times (coexistence)
inhomogeneous emission allows for several modes at the same time	each species searches (or creates) its ecological niche