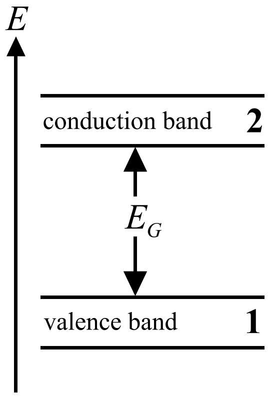
4.4 The semiconductor LASER



- In a semiconductor LASER we do not have two single energy levels but two energy bands as LASER niveaus
- Only light with $h\nu > E_G$ can be absorbed in a semiconductor. So for the stimulated emission only this frequencies occur.
- Let the rates for absorption, spontaneous and stimulated emission be

$$Z_{12}^{(abs)},\ Z_{21}^{(spot)},\ \text{and}\ Z_{21}^{(ind)} \tag{4.23}$$

- Some additional background information you can find in the semiconductor script.

Transitions occur only from occupied states into unoccupied states. We therefor find

$$\begin{aligned} Z_{12}^{(abs)} &= Kf(E_1)(1-f(E_2))\varrho(\omega) \\ Z_{21}^{(ind)} &= Kf(E_2)(1-f(E_1))\varrho(\omega) \end{aligned} \tag{4.24}$$

We find the same proportionality factor K for both processes since they are induced by quantum mechanics. (It is the same reason why $B_{12} = B_{21}$ holds).
In case of LASERing the spontaneous emission can always be neglected in comparison to the induced emission. Light amplification of a frequency ω therefor only occurs if

$$Z_{21}^{(ind)} > Z_{12}^{(abs)} \quad . \tag{4.25}$$

As LASER condition we find consequently

$$f(E_2)(1-f(E_1)) > f(E_1)(1-f(E_2)) \quad . \tag{4.26}$$

In thermodynamic equilibrium always holds

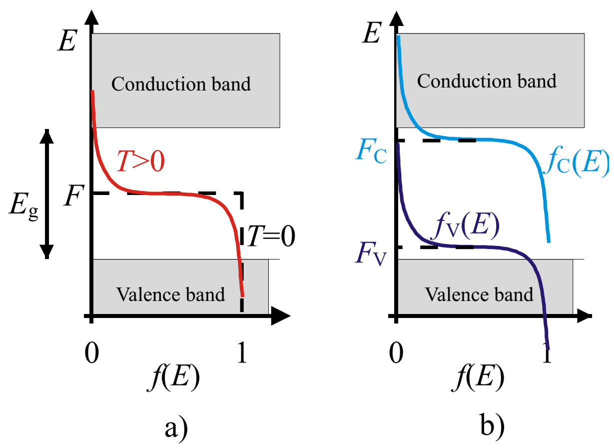
$$f(E_2) \ll f(E_1) \text{ and } (1-f(E_1)) \ll (1-f(E_2)) \quad . \tag{4.27}$$

The LASER in action therefor describes a state of extreme non equilibrium.

Description of non equilibrium in a semiconductor

The thermodynamic equilibrium in a semiconductor is described by the Fermi energy and the temperature dependent shape of the Fermi statistics:

- all electrons in all band are in thermal equilibrium
- the relative occupation probabilities depend only on the energy differences in the grand canonical ensemble
- the time which is necessary to reach the equilibrium depends on the slowest process
- the slowest process has the smallest interaction probability



Excitation times:

Atomic transitions:	$10^{-15} - 10^{-18}s$
Intra band transitions:	$10^{-8} - 10^{-12}s$
Band-band transitions:	$10^{-3} - 10^{-7}s$

$\Rightarrow$ after a time of ca. $10^{-8}s$ the occupation probabilities of electrons within a band follow the Fermi statistics.

- For one band the chemical potential exists which defines the electron number in this band
- Different bands are not in thermal equilibrium
- For different bands we need different chemical potentials
- Electrons in different bands are handled as distinguishable particles
- As for chemical reactions each type of particles has its own chemical potential
- The interaction between distinguishable particles is explicitly written as reaction equations
- In out case we explicitly name the band-band interactions to describe non equilibrium states

The Fermi statistics for the conduction band is

$$f_C(E) = \frac{1}{\exp\left(\frac{E-F_C}{kT}\right)+1} \tag{4.28}$$

For the valence band we find

$$f_V(E) = \frac{1}{\exp\left(\frac{E-F_V}{kT}\right)+1} \tag{4.29}$$

For the LASER condition we get

$$f_C(E_2)(1-f_V(E_1)) > f_V(E_1)(1-f_C(E_2)) \qquad , \tag{4.30}$$

i.e.

$$\frac{1}{\exp\left(\frac{E_2-F_C}{kT}\right)+1}\,\frac{\exp\left(\frac{E_1-F_V}{kT}\right)}{\exp\left(\frac{E_1-F_V}{kT}\right)+1} > \frac{1}{\exp\left(\frac{E_1-F_V}{kT}\right)+1}\,\frac{\exp\left(\frac{E_2-F_C}{kT}\right)}{\exp\left(\frac{E_2-F_C}{kT}\right)+1} \qquad . \tag{4.31}$$

This is equivalent to

$$F_C - F_V > E_2 - E_1 \geq E_G \qquad . \tag{4.32}$$