

3.8 The Einstein Model

For $\omega = \omega_0$ we find for the inner energy

$$U = 3 \int d\omega D(\omega) \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1} = \frac{\hbar\omega_0}{\exp\left(\frac{\hbar\omega_0}{kT}\right) - 1} 3N \quad . \quad (3.43)$$

and:

$$C_V = \frac{dU}{dT} = \frac{3N\hbar\omega}{\left(e^{\frac{\hbar\omega}{kT}} - 1\right)^2} \frac{e^{\frac{\hbar\omega}{kT}} \hbar\omega}{kT} \quad . \quad (3.44)$$

For high temperatures we again get:

$$C_V = 3Nk \quad . \quad (3.45)$$