3.2 Calculation of the inner energy

From Eq. (2.45) we know the grand canonical potential

$$\Omega(T,V,\mu) = \pm kT \sum_{\alpha} \ln\left(1 \mp \exp\left(-\frac{\epsilon_\alpha - \mu}{kT}\right)\right) \tag{3.4}$$

α indicates independent states, the plus/minus sign depend on the particles to be Fermion or Bosons. For a continuous system the sum changes into an integral:
Let

$$F_{\beta\mu}(\epsilon) := \pm kT \ln\left(1 \mp \exp\left(-\frac{\epsilon - \mu}{kT}\right)\right) \tag{3.5}$$

and

$$f_{\beta\mu}(\epsilon) := \frac{\partial F_{\beta\mu}(\epsilon)}{\partial \epsilon} = \frac{1}{\exp\left(\frac{\epsilon-\mu}{kT}\right) \mp 1} = -\frac{\partial F_{\beta\mu}(\epsilon)}{\partial \mu} \qquad . \tag{3.6}$$

Obviously we find

$$\Omega(T,V,\mu) = \int_{-\infty}^{+\infty} D(\epsilon) F_{\beta\mu}(\epsilon) d\epsilon \qquad , \tag{3.7}$$

$$N(T,V,\mu) = \int_{-\infty}^{+\infty} D(\epsilon) f_{\beta\mu}(\epsilon) d\epsilon \qquad , \tag{3.8}$$

and

$$E(T,V,\mu) = \int_{-\infty}^{+\infty} \epsilon D(\epsilon) f_{\beta\mu}(\epsilon) d\epsilon \qquad . \tag{3.9}$$

$D(\epsilon)$ is the density of states.