Basic *IU*-Characteristics of Solar Cells

Foreword

This module is at once the solution of an exercise and a detailed discussion of the *UI*-characteristics of a theoretical and practical *ideal* solar cell.

- If you are not already familiar with the diode equation and what exponential terms can do, you will profit very much by going through it in detail.
- For easy readability parts of the data given in the exercise will be repeated her in a somewhat modified form.

Starting Point

The diode equation with generation and recombination in the space charge region part describes a **pn**-junction made from a semiconductor like **Si** that has well defined properties, for example the doping concentration and the diffusion length / life time. It thus also describes an "ideal" solar cell. We will now try to see what we can do with this equation with respect to solar cells.

- We use the abbreviations j_1 and j_2 for the current densities in the brackets and $d(U)$ as the "abbreviation " for the equation shown below that gives us the width of a space charge region in a **pn**-junction.
- For the current density $j(U)$ and the (less important) space charge region width $d(U)$ we have

$$j(U) = \left(\frac{e \cdot L \cdot n_i{}^2}{\tau \cdot N_A} + \frac{e \cdot L \cdot n_i{}^2}{\tau \cdot N_D} \right) \cdot \left(\exp \frac{eU}{kT} - 1 \right) + \left(\frac{e \cdot n_i \cdot d(U)}{\tau} \right) \cdot \left(\exp \frac{eU}{2kT} - 1 \right) - j_{Ph}$$

$$(\quad\quad j_1 \quad\quad) \quad\quad\quad (\quad j_2 \quad)$$

$$d(U) = \left(\frac{2 \cdot \epsilon_{Si} \cdot \epsilon_0 \cdot (\Delta E_F - e \cdot U_{ex})}{e^2 \cdot N_D} \right)^{1/2}$$

In evaluating this equations for solar cells as schematically given in the drawing, we must first find numerical values for important parameters (all others have their usual meaning and values). We have:
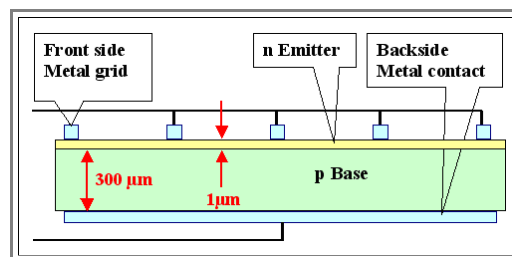
- **L = diffusion length = $(D\tau)^{1/2}$** average distance a minority carrier travels between its birth by a generation event (mostly caused by light in a "working" solar cell) and its death by recombination. A suitable value for good bulk **Si** is **L = 100 μm**.
- **D** is the diffusion coefficient and τ the (minority carrier) life time. A good enough value for the life time going with a diffusion length of **100 μm** is **τ = 1 ms**.
- **n_i** is the intrinsic carrier concentration. It increases exponentially with temperature *T*. A good value for **Si** at room temperature (**RT**) is **n_i(RT) = 10^{10} cm^{-3}**.
- **N_A** and **N_D** are the acceptor and donor concentrations in the **p**-part called **base**, the usually several **100 μm** thick part of a bulk **Si** solar cell, and the **n**- part, called **emitter**, the thin "layer" on top of the solar cell, respectively. The base is lightly doped (otherwise the diffusion length suffers) whereas the emitter is heavily doped (good conductivity is important). **N_A = 10^{16} cm^{-3}** and **N_D = 10^{19} cm^{-3}** are good round numbers for the purpose here.
- We could also calculate the width of the space charge region with the equation given but take it as **$d(U)$ = 1 μm** as a first simple approximation.

Now we consider a *real* but still "ideal" solar cell under "standard" illumination. This gives us the following (simplified) second set of numbers:

- Area of the **Si** bulk solar cell = **100 cm^2**. It's actually more like **200 cm^2** in **2008** but let's stay with easy

numbers.
- Photo current density $j_{Ph} = 30$ mA/cm^2 at "*AM 1.5*" the condition of maximum solar intensity on earth ("High noon in the tropics")
- The photo current here is thus $j_{Ph} = 3$ A.

1a: Using only the first term in the bracket for j_1 as a sufficient approximation, give an equation for the relation of j_2/j_1 and some numbers for these current densities.

1b: Does the result imply that you can neglect one of the j_i terms in the equation above in the *forward* direction? How about the *reverse* direction?

For the numerical values of j_1 and j_2 we obtain

$$j_1 = \frac{e \cdot L \cdot n_i^2}{\tau \cdot N_{dot}} = \frac{1.6 \cdot 10^{-19} \cdot 10^{-2} \cdot 10^{20}}{10^{-3} \cdot 10^{16}} \frac{C}{s \cdot cm^2} = 1.6 \cdot 10^{-14} \text{ A/cm}^2$$

$$j_2 = \frac{e \cdot n_i \cdot d(U)}{\tau} = \frac{1.6 \cdot 10^{-19} \cdot 10^{10} \cdot 10^{-4}}{10^{-3}} \frac{C}{s \cdot cm^2} = 1.6 \cdot 10^{-10} \text{ A/cm}^2$$

For the relation of j_1 to j_2 (and using N_{Dot} instead of $N_{A, D}$) we obtain

$$\frac{j_1}{j_2} = \frac{\dfrac{e \cdot (n_i)^2 \cdot L}{\tau \cdot N_{Dot}}}{\dfrac{e \cdot n_i \cdot d(U)}{\tau}} = \frac{n_i \cdot L}{N_{Dot} \cdot d(U)}$$

Inserting the numbers from above yields once more

$$\frac{j_1}{j_2} = \frac{10^{10} \cdot 10^{-2}}{10^{16} \cdot 10^{-4}} = 10^{-4}$$

Now we can address the second part of the first question. Since j_2 is so much larger than j_1, can we simply neglect the j_i terms in the diode equation? We should know the answers [from before](#):

For biasing in the reverse direction, we have $j_{rev} \approx j_1 + j_2 \approx j_2$ and we can indeed neglect j_1.

For the forward direction - which is the one of interest to us - we have approximately

$$j_{for} \approx j_1 \cdot \exp \frac{eU}{kT} + j_2 \cdot \exp \frac{eU}{2 \cdot kT}$$

No, we cannot neglect the j_1 term "just so", we also have to consider what the exponential terms will do. That will become very clear as soon as we look at the third part of the question.

The prelude to the second question was: If we now *measure* the actual **UI** characteristics of a good *real* solar cell and fit the curve obtained to our equation from above, we find values for the current densities j_1 and j_2 like

- $j_1 = 10^{-9}$ **A/cm^2**.
- $j_2 = 10^{-7}$ **A/cm^2**

The *second question* was:

🔵 **2a:** Do the measured values of j_1 and j_2 and their relation meet your expectations based on your results from *question 1*?

🔵 **2b:** If not, what could be reasons for the discrepancy?

Let's compare what we have in a table:

	Calculated	**Measured**
j_1	$1.6 \cdot 10^{-14}$ A/cm^2	10^{-9} A/cm^2
j_2	$1.6 \cdot 10^{-10}$ A/cm^2	10^{-7} A/cm^2
$\dfrac{j_1}{j_2}$	10^4	10^2

🔵 We neglected the second term for j_1, which we will now call $j_1{}^E$ for the time being; i.e. the reverse current flowing from the heavily doped thin **n**-emitter into the lightly doped **p**-base. If we would naively calculate $j^E{}_1$, we would get an even smaller value than what we already have for $j_1{}^B$ because the doping concentration N_D of the emitter is larger than N_A of the base and appears in the *denominator* of the equation for $j_1{}^E$.

🔵 However, we would commit a grave error in doing this because the diode "master" equation from above is only valid for one-dimensional junctions in "infinitely" long semiconductors, meaning that the semiconductor must extend at least a few diffusion lengths in both directions as seen from the junction. This is clearly not the case here.

🔵 More advanced theory teaches us that in the case of "thin" semiconductors we have to replace the diffusion length L by the thickness d of the layer.. This makes sense because the diffusion length came into the equation as the dimension over which carriers are collected that could make it across the junction.

🔵 If we use this insight, however, $j_1{}^E$ gets even smaller because L is found in the *nominator* of the equation.

A first but *wrong* conclusion could be the discrepancy between theory and experiment cannot come from the emitter part of the reverse current.

🔵 However, we forgot the life time τ, which we find in the *denominator* of the equation, and now we must take into account that heavy doping simply "kills" the life time, i.e. makes it very small. The diffusion length L gets smaller, too, but the combined effect is that $L/\tau \propto \tau^{-1/2}$ so heavy doping always increases the j_1 part coming from the heavily doped region, and this increase can be substantial

As a first insight we note that a heavily doped thin emitter can indeed lead to a substantial increase in j_1. But there are more reasons for this.

🔵 At the most elementary level of deriving j_1 we simply equated it with diffusion length L times the generation rate G; and G was equal to the recombination rate $R = n_{min}/\tau$. An increased j_1 thus demands for an increase in generation -we simply need more charges to have larger currents.

🔵 In not-so-perfect **Si** we might have generation of carriers at grain boundaries or at the huge surfaces in excess of what just thermal generation can produce in a perfect lattice. To be sure, the recombination rate R must still be equal to G in equilibrium, but j_1 will go up with increasing generation anyway.

We see that there are several reason why we have the discrepancy. We simply must accept that the experimental j_1 and j_2 values are essentially **fitting parameters** of something called "solar cell" that do not fall within the range of a simple theory but still allow to describe the solar cell by the "simple" ideal theory if one accepts these empirical "fitting parameters" instead of the theoretical constructs in the equation.

Now to the *third question*: Given the measured j_i values from above and the j_{Ph} value given, we now can consider the short circuit current I_{SC} and the open circuit voltage U_{OC}

Question 3:

3a: What do you get for I_{SC}? Does it depend on j_i and j_2? If not, what determines its value?

3b: What can you say about the open circuit voltage U_{OC}?

The *first* part is easy: I_{SC} is what we get for $U = 0$ and that is simply $-j_{Ph}$.

- We have used $-j_{Ph}$ as a constant in the master equation, it thus does not depend on the values of j_1 or j_2 or on the variables determining their numerical values.

- This is not generally correct, of course. For example, if the diffusion length L increases, more carriers generated by light deep in the volume of the solar cell can reach the junction and $|j_{Ph}|$ should increase with L.

- However, we have assumed *good* solar cells along, and this means that practically all carriers generated by light end up in the photo current. This simply implies that for diffusion lengths good enough not much can be gained anymore by increasing L. In other word, if the longest distance between a generation event and the next contact is **20 μm**, it just doesn't matter much if your carriers could go **200 μm** or **500 μm**.

The *second* part is tough: If we try to solve the master equation for U_{OC}, i.e. setting $j = 0$, we realize that it can't be done.

- There is no analytical expression for U_{OC} that we can gain from the master equation. Short of going numerical, we need to consider other ways of gaining some insight, including approximations.

- One way is to go for a graphical solution of the problem. We actually have done that already, but probably not recognized what we can learn for solar cells from this. All we have to do is to draw the master equation in a **log j - eU** plot. This is actually a very good exercise and you should do - at least look at the solution - and learn how it's done.

> **Exercise 8_1_6**
> **log j - eU**

The result looks like this:
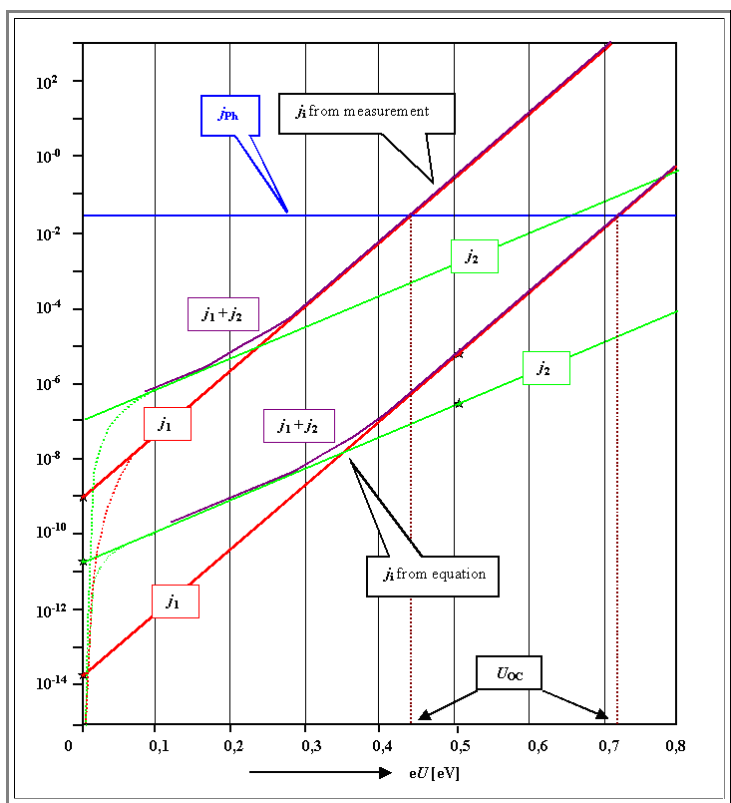
From looking at the graph we can learn a lot of things

1. The "**-1**" term in the master equation is only noticeable for currents $<\approx j_2$.

⬤ We can safely neglect it for solar cells as long as we have a photo currents in just the $\mathbf{\mu A/\,cm^2}$ region, i.e **1 000** times smaller than the maximum photo current density on earth.

2. The open circuit voltage U_{OC} depends *only* on j_1 for reasonable photo currents. Even so j_2 is much larger, the exponential term going with j_1 always "wins" for voltages above **0.3 V - 0.4 V**.

⬤ If we neglect the j_2 term in the master equation, we can solve it for U_{OC} and obtain for $\mathbf{e}U_{OC}$ as measured in **eV**:

$$eU_{OC} \;=\; kT \cdot \ln \frac{j_{Ph} - j_1}{j_1} \;\approx\; kT \cdot \ln \frac{j_{Ph}}{j_1}$$

⬤ This means that j_1 is the decisive term for U_{OC}, one of the prime properties of a solar cell.

Now we look at the temperature dependence of U_{OC}. From the solution of the exercise we take only one curve here:
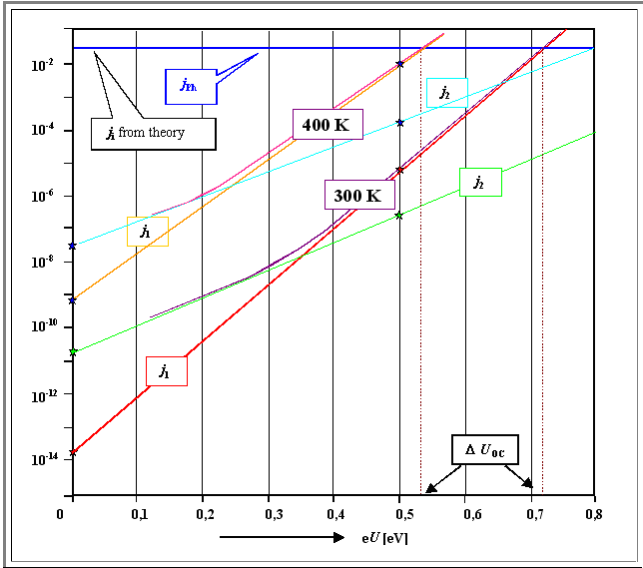
⬤ Increasing the temperature has the following effects:

1. The slope of both exponentials decreases. This would lead to a higher U_{OC}.
2. The j_i increase exponentially because their defining equations contain the intrinsic carrier density n_i, which increases exponentially with T.
3. The total effect is a decrease of U_{OC} with T

We can see that also in the equation for U_{OC}. Inserting $j_1 = c_1 \cdot n_i^2 = j_1' \cdot \exp{-(E_g/kT)}$ yields

$$eU_{OC} = E_g + kT \cdot \ln \frac{j_{Ph}}{j_1'} = E_g - kT \cdot \ln \frac{j_1'}{j_{Ph}}$$

⬤ It first looks like we *add* something to eU_{OC} with increasing T; increasing eU_{OC}. However, it is important to realize that $j_1' \gg j_{Ph}$ - even if that is counterintuitive - and $\ln(j_{Ph}/j_1')$ thus is a negative number! That the open circuit voltage indeed decreases is better seen in the final formulation where we *subtract* a positive number from E_g.

If you made it to this point, you learned quite a bit about basic solar cell characteristics. You also learned another thing:

> **Don't rely on "feeling" if**
> **exponentials**
> **are involved!**