

## 2.3.2 Quasi Fermi Energies

So far we implicitly defined (thermal) equilibrium as a *total* equilibrium involving *three* components if you think about it:

1. Equilibrium of the *electrons* in the *conduction band*. This means their density was given (within the usual approximations) by

$$n_e = N_{\text{eff}}^e \cdot \exp - \frac{E_C - E_F}{kT}$$

2. Equilibrium of the *holes* in the *valence band*. This means their density was given (within the usual approximations) by

$$n_h = N_{\text{eff}}^h \cdot \exp - \frac{E_F - E_V}{kT}$$

3. Equilibrium between the electrons and holes, i.e. *between the bands*. This means that the *Fermi energy is the same for both bands* (and positioned somewhere in the band gap).

### Definition of the Quasi Fermi Energy

If an equilibrium is disturbed, it takes a certain time before it is restored again; this is described by the *kinetics* of the processes taking place. In a strict sense of speaking, the Fermi energy is not defined without equilibrium, but only after it has been restored. This restoring process occurs in the bands and between the bands:

- In the bands*, *local* equilibrium (in *k*-space) between the carriers will be obtained after there was time for some collisions, i.e. after some multiples of the *scattering time*. This process is known as **thermalization** and occurs typically in *picoseconds*.
- Between the bands*, equilibrium will be restored by generation and recombination events and this takes a few multiples of the carrier *life time*, i.e. at least *nanoseconds* if not *milliseconds*.

This means that we can have a *partial (or local) equilibrium* in the bands long before we have equilibrium between the bands. This **local equilibrium** implies that:

- Non-equilibrium means something changes in time. Changes in the properties of the particle ensemble considered (i.e. electrons and holes) in *local* equilibrium are only due to "traffic" *between* the bands while the properties of the particles in the band do not change anymore. The term "local" of course, does not refer to a coordinate, but to a band.
- The carrier densities therefore do not have their total or global equilibrium value as given, e.g., by the *mass action law*, but their local equilibrium density can still be given in terms of the equilibrium distribution by

$$n_e = N_{\text{eff}}^e \cdot \exp - \frac{E_C - E_F^e}{kT}$$

$$n_h = N_{\text{eff}}^h \cdot \exp - \frac{E_F^h - E_V}{kT}$$

- with the only difference that the *Fermi energy now is different for the electrons and holes*. Instead of *one* Fermi energy  $E_F$  for the whole system, we now have *two* Quasi Fermi energies,  $E_F^e$  and  $E_F^h$ .

For the product of the carrier densities we now obtain a somewhat modified mass-action law

$$n_e \cdot n_h = N_{\text{eff}}^e \cdot N_{\text{eff}}^h \cdot \exp - \frac{(E_C - E_V) + (E_F^h - E_F^e)}{kT} = n_i^2 \cdot \exp - \frac{E_F^h - E_F^e}{kT}$$

- For this we used the *by now basic relation*

$$N_{\text{eff}}^e \cdot N_{\text{eff}}^h \cdot \exp - \frac{E_C - E_V}{kT} = n_i^2$$

The name "*Quasi Fermi energy*" is maybe not so good, there is nothing "*quasi*" about it. Still, that's the name we and everybody else will use. Sometimes it is also called "*Imref*" (Fermi backwards), but that doesn't help much either.

Rewriting the equations from above gives a kind of definition for the Quasi Fermi energies:

$$E_F^e = E_C - kT \cdot \ln \frac{N_{\text{eff}}^e}{n^e}$$

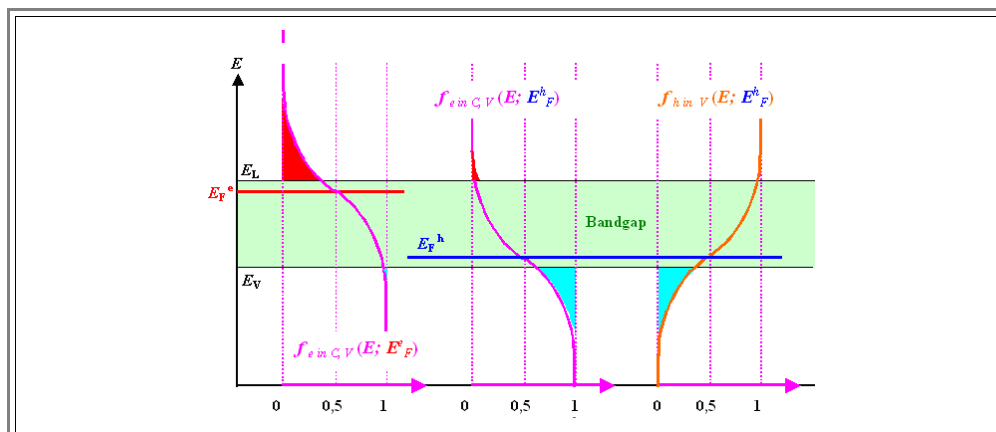
$$E_F^h = E_V + kT \cdot \ln \frac{N_{\text{eff}}^h}{n^h}$$

Quasi Fermi energies are extremely helpful for the common situation where we do have non-equilibrium, but only *between* the bands – and that covers most of semiconductor devices under conditions of current flow (due to an applied voltage) or under illumination. We will make frequent use of Quasi Fermi energies!

## Carrier Densities and Quasi Fermi Energies

If we calculate carrier densities in non-equilibrium with the Quasi Fermi energies, we have to be careful to use the right Quasi Fermi energy in the Fermi-Dirac formula or in the Boltzmann approximation.

- After all, we now have *two* (Quasi) Fermi energies, one "regulating" the density of electrons in the conduction band, and the other one doing the same for the holes in the valence band. That was already implied [above](#), here we want to make this topic a bit clearer; we also introduce a new distribution function as a kind of short-hand.
- You really must now write  $f(E, E_F^e, T)$  or  $f(E, E_F^h, T)$  instead of simply  $f(E, E_F, T)$  or  $f(E)$  because, due the different arguments, the meaning of these two expressions is now different. This is illustrated below with the two curves on the left and should be obvious.
- In the pictures we even have some redundancy by writing  $f_{e \text{ in } c}(E, E_F^e, T)$  and so on. This is not necessary, but helps in the beginning to avoid mix-up.



The density of electrons or holes in the conduction or valence band, respectively, would now be

$$n_e = N_{\text{eff}}^e \cdot [f_{e \text{ in } c}(E, E_F^e, T)] \approx N_{\text{eff}}^e \cdot \exp - \frac{E_C - E_F^e}{kT}$$

$$n_h = N_{\text{eff}}^h \cdot [1 - f_{h \text{ in } v}(E, E_F^h, T)] \approx N_{\text{eff}}^h \cdot \exp - \frac{E_F^h - E_V}{kT}$$

- The red or blue triangles above symbolize the density of electrons in the conduction band or holes in the valence band, respectively, [as before](#).
- The right-hand side is identical (of course) to what we had [above](#) and shows a kind of symmetry not contained in the formulation with the Fermi distribution, where we have  $f(E, E_F, T)$  and  $1 - f(E, E_F, T)$ .
- This can be remedied easily by simply setting  $1 - f(E, E_F, T) =: f_{h \text{ in } v}(E, E_F, T)$  with  $f_{h \text{ in } v}$  being the probability of finding holes on the available states in the valence band.
- This is the curve shown on the right-hand side in the picture above.
- If we use that definition, we obtain more symmetry at the cost of more heavily indexed functions. It's a matter of taste.
- However, later we will encounter situations where proper bookkeeping of electrons and holes is complicated and essential. Then it might be easier to keep the situation symmetric, to use  $f_{h \text{ in } v}$  for the holes in the valence band, and to express *all* carrier densities in the valence band with  $f_{h \text{ in } v}$ , while in the conduction band we use  $f_{e \text{ in } c}$ .