Semiconductors & Defects: Exercise 2 (08 Nov. '22)

General remark: Always try to come up with a short answer that catches the essence.

3. Discussion and drawing: What is the special significance of the Brillouin zones? Assuming a regular 1D chain of atoms, explain why an electron (treated in the nearly free particle approximation) exhibits two energy values at $\boldsymbol{k} = \boldsymbol{k}_{BZ}$ ($\boldsymbol{k}$: wave vector). Draw and discuss the $E(\boldsymbol{k})$ diagram for (i) the free electron gas and (ii) the free electron gas with diffraction at the first Brillouin zone edge. What is the advantage of the reduced zone scheme?

4. Formula and discussion: What is the Bloch theorem and its special significance for electrons in a crystal? For a Bloch wave, discuss the meaning of the "quasi wave vector" and the "crystal momentum."

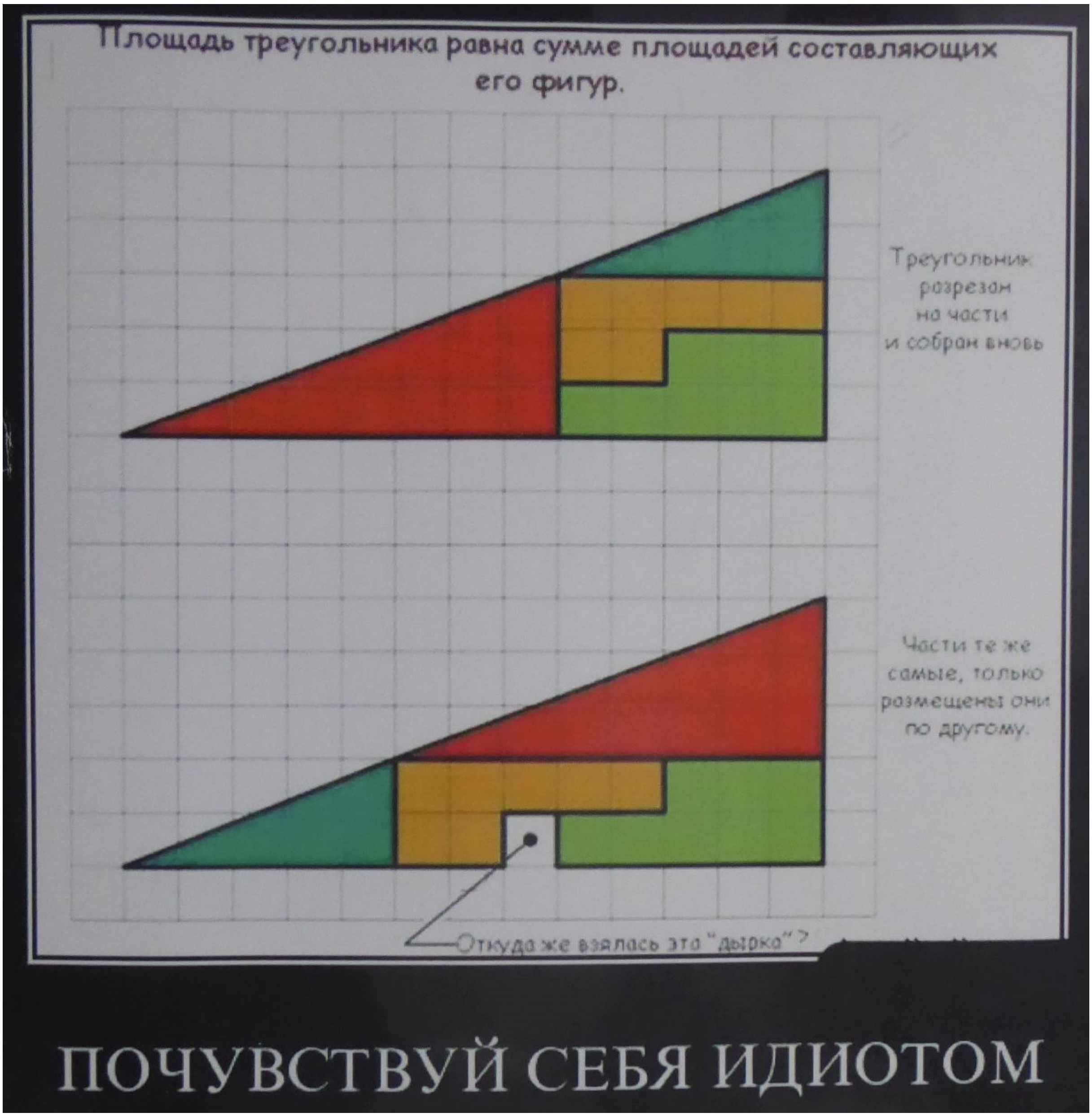
5. Calculation: Show that the width of the "soft zone" of the Fermi–Dirac distribution $f(E)$ is approximately $4\,k_BT$. [Hint: Use the tangent to $f(E)$ at $E = E_F$.]

6. Formulae and discussion: Using the relevant formula, explain how the carrier density in an energy interval depends on the density of states (DOS) and on the Fermi–Dirac distribution function. What is the difference between the true DOS function, $D(E)$, and the effective density of states, N_{eff}? For which condition is the latter a good approximation to the former? Mathematically speaking, in which way does N_{eff} simplify the equations needed to describe a semiconductor (SC)? In physics terms, how does the usage of N_{eff} simplify the thinking about the electronic transitions taking place in a SC?

7. Discussion and drawing: What is meant by an "intrinsic semiconductor"? Is it possible to have a perfect intrinsic SC in practical life? Why? Draw the schematic band diagram for conduction band, valence band, and Fermi energy for a perfect intrinsic SC. What is the "intrinsic carrier density" n_i? Using the Fermi–Dirac distribution function, express the density of electrons and holes in different bands, assuming identical densities of state in valence and conduction band. How will this distribution change if one increases the temperature? Hence, how does n_i depend on the temperature, and on the bandgap?

8. Discussion: Lift the seeming contradiction between the images shown on the second page by explaining how this geometric paradox "works."

   (Remarks: This is an off-topic task, deliberately introduced here just for to train both your ability to catch the essence as well as your logical argumentation skills. The key word in this task is "explaining" – think about what makes a statement an essence-catching explanation.)



By the way, the Cyrillic text is Russian, the line below meaning “feel like an idiot”, while the headline tells “The area of a triangle is equal to the sum of the areas of the figures that make up the triangle.” On the right it says “The triangle has been cut into pieces and reassembled again” and “The parts are the same, only they are placed differently”. Finally, there is the question “Where did the hole come from?”