6. Principles of the Semiconductor Laser

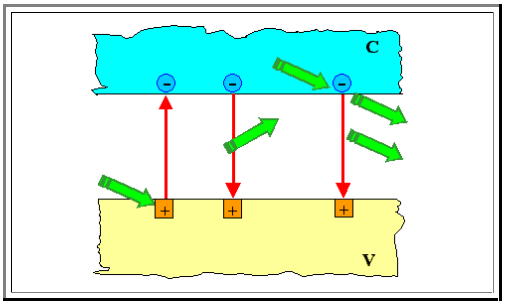
6.1 Laser Conditions

6.1.1 Interaction of Light and Electrons; Inversion

- In principle, anything that emits electromagnetic radiation can be turned into a "**LASER**", but what *is* a laser?
  - The word "*LASER*" was (and of course still is) an ***acronym***, it stands for " **L**ight **A**mplification by **S** timulated **E**mission of **R** adiation". By now, however, it is generally perceived as a standard word *in any language*, meaning something that is more than the acronym suggests (and we will therefore no longer write it with capital letters)!
  - A laser in the direct meaning of the acronym is a black box that emits (= outputs) more light of the same frequency than what you shine (= input) on it – that is the *amplifier* part. But the "*stimulated emission*" part, besides being the reason for amplification, has a second, indirect meaning, too: The light emitted is exactly in phase (or coherent to) the light in the input. Unfortunately, lasers in this broad sense do not really exist. Real lasers only amplify light with a very specific frequency – it's like electronic amplifiers for *one frequency only*.
- A laser in the *general* meaning of the acronym thus produces intense monochromatic electromagnetic radiation in the wavelength region of light (including infrared and a little ultra-violet; there are no sharp definitions) that is coherent to the (monochromatic) input. If you "input" light containing all kinds of frequencies, only one frequency becomes amplified.
  - A laser in the *specific* meaning of everyday usage of the word, however, is more special. It is a **device** that produces a coherent beam of monochromatic light *in one direction only* and, at least for semiconductor lasers, *without some input light* (but with a "battery" or power source hooked up to it). It is akin to an electronic oscillator that works by internally feeding back parts of the output of an amplifier to the input for a certain frequency.
  - Before the advent of hardware lasers in the sixties, there were already "**masers** " – just take the "**m**" for "microwave" and you know what it is.
  - And even before that, there was the basic insight or idea behind masers and lasers, and – as ever so often – it was **A. Einstein** who described the "**S**timulated **E** mission" part in **1917/1924**. More to the history of lasers can be found in an advanced module.
- Obviously, for understanding lasers, we have to consider *stimulated emission* first, and then we must look at some *feedback* mechanism.

Stimulated Emission of Radiation

- Understanding stimulated emission is relatively easy; all we have to do is to introduce one more process for the interaction between light and electrons and holes. So far we considered two basic processes, to which now a third one must be added:
  - **1. Fundamental absorption**, i.e., the interaction of a photon with an electron in the *valence band* resulting in a electron(C)–hole(V) pair.
  - **2. Spontaneous emission** of a photon by the (spontaneous and direct) recombination of an electron–hole pair.
  - **3. Stimulated emission**, as the third and new process, is simply the interaction of a photon with an electron in the *conduction band*, **forcing** recombination and thus the emission of a second photon, being an exact duplicate of the incoming one.
- All three processes are schematically shown in the band diagram below.



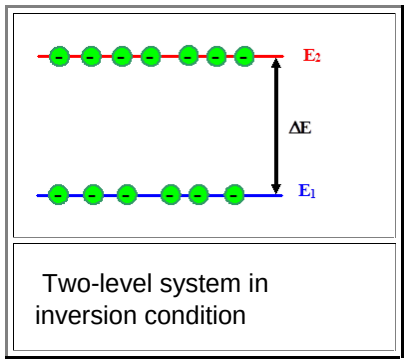
- Looking at this picture, you should wonder why one obvious further process is missing: How about an electron in the conduction band simply absorbing a photon? The electron could be moved up by the amount **hν** in the conduction band, and would come back to the band edge by tranferring its surplus energy to phonons.
  - This process does take place, but it is not very strong if we do not have many electrons in the conduction band. More importantly: It is not necessary for "lasing", but rather detrimental – we will cover it later.

- Stimulated emission, however, is *not* just the reverse of absorption. Again: Usually, photons interact with electrons in the conduction band by *transferring their energy to the electron*, which moves the electron to some higher energy level in the band (or to the next band, or, if the photons are very energetic (meaning **X**-rays), even out of the crystal) – which means that the photons are *absorbed* .
  - On the contrary, stimulated emission is a *resonant process*; it only works if the photons have exactly the right energy, corresponding to the energy that is *released* if the electron makes a transition to some allowed lower level. Then, the two photons are *exactly in phase* with each other (and propagate in the same direction). For semiconductors, this energy is pretty much the band gap energy, because all conduction band electrons are sitting at the conduction band edge (more precisely, within some small energy interval above E_C , of course), and the only available lower energy level are the free positions (i.e., occupied by holes) at the valence band edge.
  - Stimulated emission thus may be seen as a competing process to the fundamental band–band absorption process described before. But while *all* photons with an energy $h\nu > E_g$ may cause fundamental absorption, because there are many unoccupied levels above E_g, *only* photons with $h\nu = E_g$ (plus some small ΔE, possibly) may cause stimulated emission.
- Einstein showed that under "normal" conditions (meaning conditions not too far from thermal equilibrium), *fundamental absorption by far exceeds stimulated emission*. Of course, Einstein did not show that for semiconductors, but for systems with well-defined energy levels – atoms, molecules, whatever.
  - However, for the special case that a sufficiently large number of electrons occupies an excited energy state (which is called **inversion**), stimulated emission may dominate the electron–photon interaction processes. Then *two* photons of identical energy and being exactly in phase come out of the system for *one* photon going into the system.
  - The kind of *inversion* we are discussing here should not be mixed up with the *inversion* that turns **n**-type Si into **p**-type (or vice versa) that we encounterd before. Same word, but different phenomena!
  - These two photons may cause more stimulated emission – yielding **4**, **8**, **16**, ... photons, i.e. an avalanche of photons will be produced until the excited electron states are sufficiently depopulated.
  - In other words: One photon $h\nu$ impinging on a material that is in a state of *inversion* (with the right energy difference $h\nu$ between the excited state and the ground state) may, by stimulated emission, cause a lot of photons to come out of the material. Moreover, these photons are all in phase, i.e. we have now a strong and coherent beam of light – amplification of light occurred!
- We are now stuck with two basic questions:
  - **1.** What exactly do we mean with "inversion", particularly with respect to semiconductors?
  - **2.** How do we reach a state of "inversion" in semiconductors?
- Let's look at these questions separately!

Obtaining Inversion in Semiconductors

- If you shine **10** input photons on a crystal, **6** of which disappear by fundamental absorption, leaving **4** for stimulated emission, you now have **8** output photons. In the next round you have $2 \cdot (8 \cdot 0.4) = 6.4$ and pretty soon you have none.
  - Now, if you reverse the fractions, you will get **12** photons in the first round, $2 \cdot (12 \cdot 0.6) = 14.4$ the next round – you get the idea.
  - In other words, the *coherent* amplification of the input light only occurs for a *specific condition*:
- There must be *more* stimulated emission processes than fundamental absorption processes if we shine light with $E = h\nu = E_g$ on a direct semiconductor – this condition defines " *inversion* " in the sense that we are going to use it.
  - Note that the light produced by spontaneous recombination of the electron–hole pairs, generated by fundamental absorption, is not coherent to the input and does not count!
  - We only look at *direct* semiconductors, because radiative recombination is always unlikely in indirect semiconductors, and while stimulated emission is generally possible, it also needs to be assisted by phonons and thus is unlikely, too.
- We will find a rather simple relation for the dominance of stimulated emission, but it is not all that easy to derive. Here we will take a "*shortcut* ", leaving a more detailed derivation to an advanced module.

- Let's first consider some basic situations for inversion in full generality. For the most simple system, we might have two energy levels E_1 and E_2 for atoms (take any atom), the lower one (E_1) mostly occupied by electrons, the upper one (E_2) relatively empty. *Inversion* then means that the number of electrons on the upper level, n_2, is larger or at least equal to n_1.
- In equilibrium, however, we would simply have



Two-level system in inversion condition

$$\frac{n_2}{n_1} = \frac{D_2}{D_1} \cdot \exp\left(-\frac{\Delta E}{kT}\right)$$

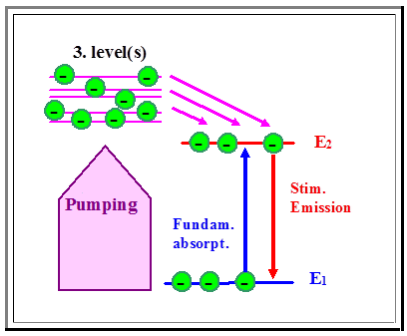
- Here, $\Delta E = E_2 - E_1$, and $D_{1,2}$ = the maximum number of electrons permitted on $E_{1,2}$ (the "density of states").
- In words: In equilibrium we have far more electrons at E_1 than at E_2 .

For inversion to occur, we therefore must be *very far from equilibrium* if ΔE is in the order of **1 eV** as needed for visible light.

- However, stimulated emission would quickly depopulate the E_2 levels, while fundamental absorption would kick some electrons back. Nevertheless, after some (short) time we would be back to equilibrium.
- To keep stimulated emission going, we must move electrons from E_1 to E_2 *by some outside energy source*. Doing this with some other light source providing photons of the only usable energy ΔE would not only defeat the purpose of the game (since, after all, that is the light we want to generate) – it also would never bring us back to inversion because of the depopulation of E_2 by stimulated emission.
- *In short*: Two-level systems are no good for practical uses of stimulated emission.
- In semiconductors we could inject electrons from some other part of the device, but a semiconductor is not a two-level system, so that is not possible.

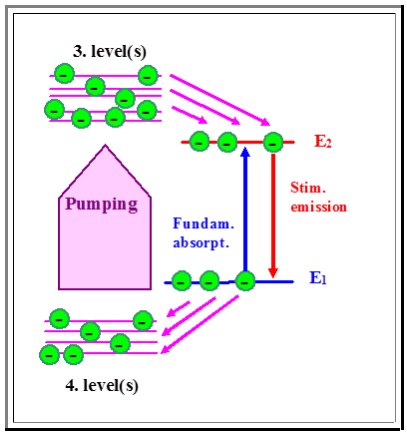
What we need is an *easy* way to move a lot of electrons to the energy level E_2 *without* depopulating it at the same time. This can be achieved in a **three-level system** as shown below (and this was the way it was done with the first ruby laser).

- The essential trick is to have a whole system of levels – ideally a band – *above* E_2, from which the electrons can descend efficiently to our single level E_2 – but not easily back to E_1 where they came from. Schematically, this looks like the figure on the right.
- The advantage is obvious. We now can use light with a whole range of energies – always larger than ΔE – to "*pump*" (yes, this is the standard word used for this process) electrons up to E_2 via the reservoir provided by the third level(s).
- The only disadvantage is that we have to take the electrons from E_1. And no matter how hard we pump, the effectiveness of the pumping depends on the probability that a quantum of the energy we pour into the system by pumping will actually find an electron to act upon. And this will always be proportional to the number (or density) of electrons available to be kicked upwards. In the three-level system this is at most D_1. However, if we sustain the inversion, it is at most $0.5 \cdot D_1$, because by definition we then have at least one-half of the available electrons on E_2.

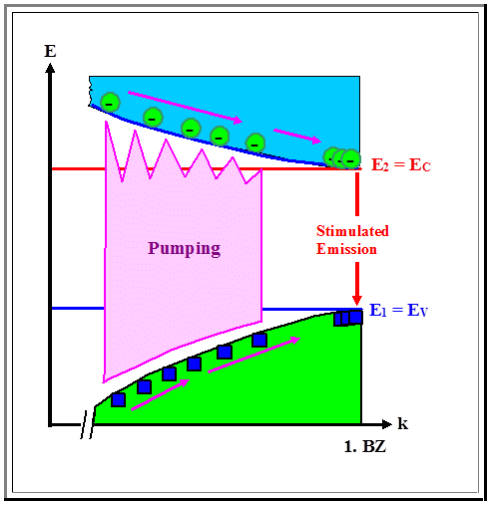


It is clear what we have to do: Provide a *fourth level* (even better: a band of levels) *below* E_1, where you have a lot of electrons that can be kicked up to E_2 via the third level(s). It is clear that we are talking semiconductors now, but let's first see the basic system:

- We simply introduce a system of energy states below E_1 in the picture from above. We now have a large reservoir to pump from, and a large reservoir to pump to.
- All we have to do is to make sure that pumping is a one-way road, i.e. that there are no (or very few) transitions from the levels **3** to levels **4**.
- This is not so easy to achieve with atoms or molecules, but, as you should have perceived by now, this is exactly the situation that we have in many direct band gap semiconductors. All we have to do to see this is to redraw the **4**-level diagram at the right as a band diagram. To include additional information, we do this in *k-space*.



We have the following general situation for producing inversion in semiconductors by optical pumping:



Electrons may be pumped up from anywhere in the valence band to anywhere in the conduction band – always provided the transition goes vertically upwards in the reduced band diagram.

- The electrons in the conduction band as well as the holes in the valence band will quickly move to the extrema of the bands – corresponding to the levels E_2 and E_1 in the general four-level system.
- "*Quickly*" means within a time scale defined by the dielectric relaxation time. This time scale is so small indeed that it introduces some uncertainties in the energies via the **uncertainty relation** (which is considered in the advanced module but need not bother us here).

We have now everything needed for a "quick and dirty" derivation for the inversion condition in the sense introduced at the top.

The Inversion Condition

The condition for inversion was that there were at least as many stimulated emission processes as fundamental absorption processes. The recombination rate by **stimulated emission** we now denote R_{se}, and the electron–hole pair generation rate by **fundamental absorption** is R_{fa}. We thus demand:

$$R_{se} \geq R_{fa}$$

- In general, fundamental absorption and stimulated emission can happen in a whole range of frequencies for semiconductors. While we expect that the electrons that are being stimulated to emit a photon will occupy levels right at the conduction band edge, stimulated emission is not forbidden for electrons with a higher energy somewhere in the conduction band. While these electrons are in the (fast) process of relaxing to E_C , they still might be "hit" by a photon of the right energy at the right time and place – it is just more unlikely than at E_C.

We thus must expect both rates, R_{se} and R_{fa}, to be proportional to:

1. The **spectral intensity of the radiation** in the interesting frequency interval.
   - The differential frequency interval considered extends from ν to $\nu + \Delta\nu$; the spectral intensity in this interval we name $u(\nu)\Delta\nu$ or, expressing the frequency ν in terms of energy via $E_{phot} = h\nu$, $u(E)\Delta E$.
   - This value, $u(E)\Delta E$, divided by the single-photon energy $E_{phot} = h\nu$, essentially gives *the flux of photons in this frequency interval* (i.e., the number of photons arriving per second and per area, for short).

2. The **density of states** available for the processes.
   - The probability that a photon with a certain frequency ν and therefore energy $E_{phot} = h\nu$ will be absorbed by an electron at *some* position E_1 in the valence band (i.e., close but not necessarily equal to E_V), will be proportional to the density of states in the valence band, $D_V(E_1)$, *and* to the density of states exactly E_{phot} above this position in the conduction band, $D_C(E_1 + h\nu)$.
   - Contrariwise, the probability that stimulated emission takes place, triggered by a photon with energy $h\nu$, is proportional to the density of states in the conduction band *and* to the density of states $h\nu$ below in the valence band.
   - This is a crucial part of the consideration – and a *rather strange one*, too: That *both* densities of states must be taken into account – where the particle is coming from *and* where it is going to – is a quantum mechanical construct (known as Fermi's golden rule) that has no classical counterpart.
3. The **probability** that the states are actually occupied (or unoccupied).
   - The density of states just tells us how many electrons (or holes) *might* be there. The important thing is to know how many *actually are there* – and this is given by *the probability that the states are actually occupied* (necessary for absorption or stimulated emission) or *unoccupied* (necessary for the transition of electrons to this state).
   - In other words, the *Fermi–Dirac distribution* comes in. In the familiar nomenclature we write it as $f(E, E_F^{e}, T)$ or $f(E, E_F^{h}, T)$ with $E_F^{e,h}$ = quasi Fermi energy for electrons or holes, respectively.
   - The *crucial* point is that we take the *quasi Fermi energies*, because we are *by definition* treating strong non-equilibrium between the bands, but (approximately) equilibrium in the bands.
   - We also, for ease of writing, define a direct Fermi distribution for the holes as outlined before and distinguish the different distributions by the proper index:

$f_{e \text{ or } h}(E, E_F^{e,h}, T)$	=	probability that some level at energy E *is* occupied by an electron or hole
$1 - f_{e \text{ or } h}(E, E_F^{e,h}, T)$	=	probability that some level at energy E is *not* occupied by an electron or hole

   - Remember that "not occupied by a hole" always means "occupied by an electron" – whereas the meaning of "not occupied by an electron" depends on what is referred to: Only for the valence band this means "occupied by a hole"! (Do you also remember why this is so? If not: Think about charge neutrality!)

That is all. However, the density of states are complicated functions of E, and the spectral density of the radiation we do not know – it is something that should come out of the calculations.

- But we are doing shortcuts here, and we do know that the radiation density will have a maximum around $h\nu = E_g = E_C - E_V$. So let's simply assume that the necessary integrations over $u(E) \cdot D(E)\Delta E$ will be expressible as $N_{eff} \cdot u(\nu) \cdot \Delta\nu$ with N_{eff} = effective density of states. Moreover, we assume identical N_{eff} in the valence and conduction band.
- The rates R_{se} for stimulated emission and R_{fa} for fundamental absorption then can be written as

$$R_{fa} = A_{fa} \cdot N_{eff}^2 \cdot u(\nu) \cdot \Delta\nu \cdot \left(1 - f_{h \text{ in } V}(E_1, E_F^{h}, T)\right) \cdot \left(1 - f_{e \text{ in } C}(E_1 + h\nu, E_F^{e}, T)\right)$$

$$R_{se} = A_{se} \cdot N_{eff}^2 \cdot u(\nu) \cdot \Delta\nu \cdot \left(f_{e \text{ in } C}(E_1 + h\nu, E_F^{e}, T)\right) \cdot \left(f_{h \text{ in } V}(E_1, E_F^{h}, T)\right)$$

- The A_{fa} and the A_{se} are the proportionality coefficients and we always use $f_{h \text{ in } V}$ if we consider carriers in the valence band and $f_{e \text{ in } C}$ if we consider the conduction band.

Enters Albert **Einstein**. He showed in **1917** that the following extremely simple relation *always* holds for fundamental reasons:

$$A_{fa} = A_{se}$$

- We will just accept that (if you don't, turn to the advanced module for a derivation) and now form the ratio R_{se} / R_{fa}. The coefficients then just drop out and we are left with

$$\frac{R_{se}}{R_{fa}} = \frac{[f_{e \text{ in } C}(E_1 + h\nu, E_F^{e}, T)] \cdot [f_{h \text{ in } V}(E_1, E_F^{h}, T)]}{[1 - f_{h \text{ in } V}(E_1, E_F^{h}, T)] \cdot [1 - f_{e \text{ in } C}(E_1 + h\nu, E_F^{e}, T)]}$$

- With some shuffling of the terms (see the exercise below) we obtain

$$\frac{R_{se}}{R_{fa}} = \frac{E_F^e - E_F^h}{h\nu}$$

- with E_1 and $E_1 + h\nu$ denoting some energy level in the valence or conduction band, respectively, implying $h\nu \geq E_g$ (since for direct semiconductors, the smallest possible difference between some energy levels in the valence band and some energy levels in the conduction band that are connected by a direct transition is E_g).

This is a rather simple, but also rather important equation. It says that we have *more* stimulated emission between $E_1 + h\nu$ and E_1 than fundamental absorption between E_1 and $E_1 + h\nu$ if the *difference in the quasi Fermi energies is larger than the difference between the considered energy levels*.

- Thus, we have as the **first laser condition**:

$$E_F^e - E_F^h \geq h\nu \geq E_g$$

- We call this "*laser condition* ", because "lasing" requires inversion, i.e. that there are at least as many electrons at the conduction band edge as we have *electrons* (not holes!) at the valence band edge.

It is clear that this involves heavy non-equilibrium conditions.

- We need to *inject a lot of electrons* into the conduction band and a *lot of holes* (= taking electrons out) into the valence band.
- And we have to keep the *injection rates* at least as large as the stimulated emission rate, i.e. we have to supply electrons (and holes) just as fast as stimulated emission takes them away if we want to keep the rate of radiation constant.

Now we know what is needed to obtain light amplification in principle. But how much amplification do we get from a piece of semiconductor kept in inversion? This will be the topic of the next module.

Exercise 6.1-1

Do the math for the **1st** laser condition