Gain Coefficient

Advanced

- The **gain coefficient** describes how the density of photons, $u_\nu(z)$, changes as they propagate along the z-direction. The definition implicitly used before was

$$\frac{u_\nu(z)}{dz} = g_\nu \cdot u_\nu(z)$$

- The physical process for the change of the photon density was stimulated emission (increasing the density) and fundamental absorption (decreasing the density). Both effects we combined into a net emission rate which expresses the balance of emission or absorption rates taking place the photons propagate in z direction:

$$R^{net}{}_{se} = R_{se} - R_{fa} = R^{net}{}_{se}(z) = R(z)$$

- For the individual emission rates R_{se} and R_{fa} we had simplified equations, however, not expressively as a function of z:

$$R_{fa} = A_{fa} \cdot N_{eff} \cdot u_\nu \cdot \Delta\nu \cdot [1 - f_{h\ in\ V}(E^V, E_F{}^h, T)] \cdot [1 - f_{e\ in\ C}(E^C, E_F{}^e, T)]$$

$$R_{se} = A_{se} \cdot N_{eff} \cdot u_\nu \cdot \Delta\nu \cdot [f_{e\ in\ C}(E^C, E_F{}^e, T)] \cdot [f_{h\ in\ V}(E^V, E_F{}^h, T)]$$

- From a somewhat more detailed look at the inversion condition in an advanced module, using e.g. the proper density of states instead of effective densities, we obtained "better" equations which we are now going to use:

$$R_{fa}(E^V, E^C) = \left(A_{fa}\right) \cdot \left(D_V(E^V) \cdot \Delta E^V \cdot [1 - f(E^V, E_F{}^h)]\right) \cdot \left(D_C(E^C) \cdot \Delta E^C \cdot [1 - f(E^C, E_F{}^e)]\right) \cdot \left(u(\nu)\right)$$

$$R_{se}(E^C, E^V) = \left(A_{se}\right) \cdot \left(D_V(E^V) \cdot \Delta E^V \cdot [1 - f(E^V, E_F{}^h)]\right) \cdot \left(D_C(E^C) \cdot \Delta E^C \cdot f(E^C, E_F{}^e)\right) \cdot \left(u(\nu)\right)$$

- Summing up (= integration) for all possible transitions gives for R^{net}

$$R^{net} = A \cdot u_\nu \cdot \int_{E_C} [D_C(E^V + h\nu) \cdot D_V(E^V)] \cdot [f(E^V + h\nu, E_F{}^e) + f(E^V, E_F{}^h) - 1] \cdot dE^V$$

- The *change* in the density of the photons is now directly given by

$$\frac{\partial u_\nu(z,t)}{\partial t} = R^{net}$$

which we can write as

$$\frac{\partial u_\nu(z,t)}{\partial t} = \frac{\partial u_\nu(z,t)}{\partial z} \cdot \frac{\partial z}{\partial t} = R^{net}$$

- We use the partial derivative signs ∂ to make clear that we have more than one variable.

- This may look at bit strange. What does it mean?

- It means that the density of a bunch of photons that are contained in some volume element at some point ***z*** is given by the product of the change in density along ***z*** that they experience in their travel, times the rate with which they change their position and this means that

$$\frac{\partial z}{\partial t} = v_g = \text{group velocity of the photons .}$$

Look at a simple analogy:

- When you and your friends travel as a group from Kiel to Munich, starting with some amount of money m_{Kiel} , which will cerainly change by the time you reach Munich, you have a certain value of the money *gradient* **d*m*/d*l*** along the length ***l*** of your path.
- Your *rate of spending*, **d*m*/d*t*** , depends on how *much* you spent along the way (**= d*m*/d*l***) times how *fast* you spent it (**= d*l*/d*t***),

$$\frac{dm}{dt} = \frac{dm}{dl} \cdot \frac{dl}{dt}$$

- and **d*l*/d*t*** is just the velocity with which you move.

For $\{\partial u_\nu(z,t)/\partial z\} \cdot \{\partial z/\partial t\}$ we already have the independent expression that defined the gain coefficient from above, and we also have the lengthy expression for R^{net}. Inserting it yields

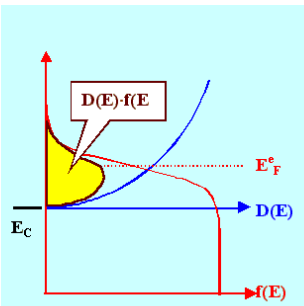
$$R^{net} = g_\nu(z) \cdot v_g \cdot u_\nu = A \cdot u_\nu \int_{E_C} \Big(D_C(E^v + h\nu) \cdot D_V(E^v)\Big] \cdot \Big(f(E^v + h\nu, E_F^e) + f(E^v, E_F^h) - 1 \Big) \cdot dE^v$$

- from which we obtain the final formula

$$g_\nu = \frac{A}{v_g} \cdot \int_{E_C} \Big(D_C(E + h\nu) \cdot D_V(E^v)\Big) \cdot \Big(f(E^v + h\nu, E_F^e) + f(E^v, E_F^h) - 1 \Big) \cdot dE^v$$

This looks complicated (actually, it *is* complicated) - but it is a clear recipe for calculating ***g***.

- Essentially, the integral as a function of the frequency ν scales with the density of electrons in the conduction band and the density of holes in the valence band exactly **hν** electron volts below. Both values increase if the Quasi Fermi energies move deeper into the bands.
- The integral runs over the valence band, summing up all energy couples between the valence band and the conduction band that are separated by **hν** ; it will thus be a function of ν. For some ν , depending on the carrier concentration, it will have a maximum. This is easy to see if we consider the distribution of electrons (or holes) in the conduction (or valence) band.



- In this example for the conduction band, the quasi Fermi energy is somewhere above the band edge. The product of the Fermi distribution with the density of states (here as the standard parabola from the free electron gas approximation) always will give a pronounced maximum somewhere between E_C and E^e_F. The same thing happens for the holes in the valence band.
- The energy difference between the two maxima will be the energy or frequency where g_V is largest. If we increase the carrier concentrations, i.e. if we move the quasi Fermi energies deeper into the bands, g_V will increase too, and the maximum value shifts to somewhat larger energies.

All things considered, we now have:

- A good idea of how to calculate g_V and what we need to know for the task.
- A good idea of the general behavior of g_V and what we have to do in a qualitative way to change its value to what we want.
- A pretty good grasp why g_V looks the way we have drawn it - without justification - in a backbone module.