5.4 Quantum Devices

5.4.1 Single and Multiple Quantum Wells

Energy Levels in a Single Quantum Well

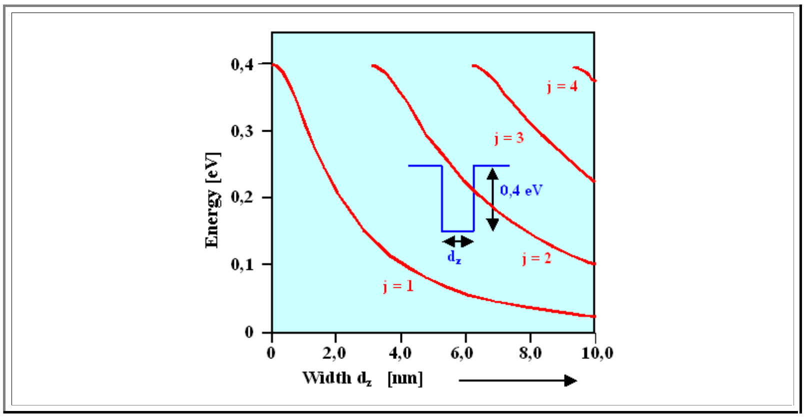
- Let's first look at an ideal **single quantum well** (**SQW**), rectangular and with an extension d_z and infinite walls (the index "**z**" serves to remind us that we always have a three-dimensional system with the one-dimensional quantum structures along the ***z***-axis).
  - We have already solved the Schrödinger equation for this problem: It is nothing else but the one-dimensional free electron gas with d_z instead of the length ***L*** of the crystal used before.
  - We thus can take over the solutions for the energy levels; but being much wiser now, we use the effective mass instead of the real mass for the electrons and obtain

$$E = \frac{(\hbar \cdot k_z)^2}{2m_z^*}$$

  - With $k_z = \pm n_z \cdot 2\pi/d_z$, and **n = ± (0,1,2,3,...)**.
  - We have used *periodic boundary conditions* for this case, which is physically sensible for large crystals. The wave functions are propagating plane waves in this case. It is, however, more common and sensible to use fixed boundary conditions, especially for small dimensions. The wave functions then are standing waves. Both boundary conditions produce identical results for energies, density of states and so on, but the set of wave vectors and quantum numbers are different; we have
  - $k_z = j_z \cdot \pi/d_z$, and $j = 1,2,3,..$ (we use ***j*** as quantum number to indicate a change in the system). For the energy levels in a single quantum well we now have the somewhat modified formula

$$E = \frac{\hbar^2 \cdot \pi^2}{2m_z^*} \cdot \frac{j^2}{d_z^2}$$

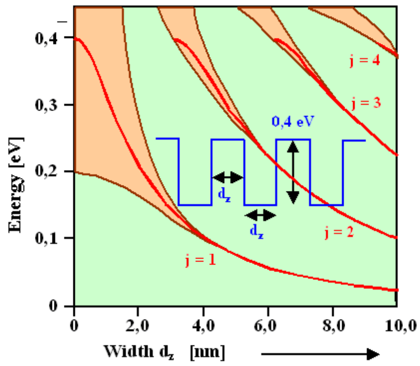
- The absolute value of the energy levels and the spacing in between increases with decreasing width of the **SQW**, i.e. with decreasing thickness d_z of the small band gap semiconductor sandwiched between the two large band gap semiconductors.
  - Large differences in energy levels might be useful for producing light with interesting wave lengths. In infinitely deep ideal **SQWs** this is not a problem, but what do we get for real **SQWs** with a depth below **1 eV**? This needs more involved calculations, the result is shown in the following figure.



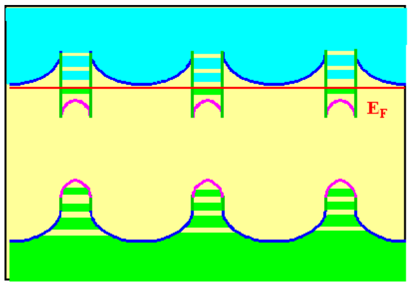
  - The **SQW** has a depth of **0.4 eV**; if it disappears for $d_z = 0$, we simply have a constant energy of **0.4 eV** for the ground state; all excited states stop at that level.
- For layer thicknesses in the **nm** region (which is technically accessible) energy differences of **0.2 . . . 0.3 eV** are possible, which are certainly interesting, but not so much for direct technical use.
  - While **SQWs** are relatively easy to produce and provide a wealth of properties for research (and applications), we will now turn to *multiple* quantum wells obtainable by periodic stacking of different semiconductors as shown before.

Energy Bands in Multiple Quantum Wells

- Since single atoms may also be described as **SQWs** (for one electron you just have the hydrogen atom type with a Coulomb potential), we must expect that the wave function of the electrons start to overlap as soon as the single **SQWs** in the **MQW** structure are close enough.
  - The situation is completely analogous to the qualitative formation of a crystal with a periodic potential from atoms. The discrete energy levels must split into many level, organized in bands.
  - This is exactly what happened; it was **Leo Esaki** who first did the calculations (and more) for this case for which he was awarded the Nobel prize.
  - Her is the result for the same system shown above.



  - Energy bands with respectable band gaps develop indeed, and we now should redraw the band diagram from before to include this fact:



  - We have "mini-bands" in the quantum wells (for symmetry reasons also for the holes in the quantum well along the valence band); green denotes occupied levels; blue empty levels.
- What do we gain by this (besides a Nobel prize)?
  - *A structure that is used in commercially sold LASER diodes!* In other words, quite crucial parts of the information technology rely on **MQWs**.
  - We will come back to this issue in the context of semiconductor lasers.