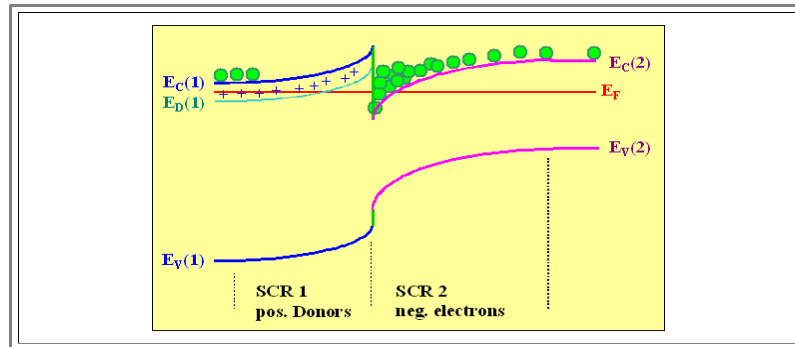


5.3.2 Isotype Junctions, Modulation Doping, and Quantum Effects

Isotype Junctions

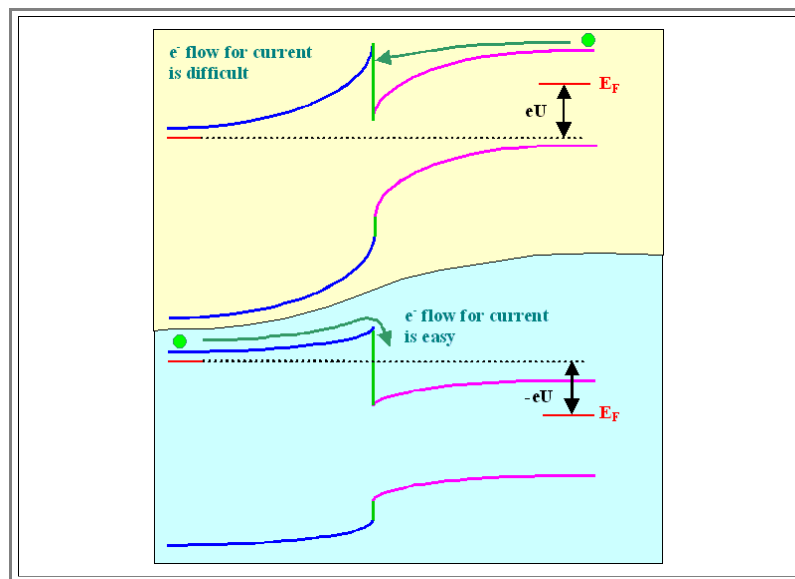
Let's look at the band diagrams of an isotype heterojunction in equilibrium; we chose a somewhat extreme case:



- Electrons were transferred from the (heavily **n**-doped) wide-gap material **1** to the (lightly **n**-doped; almost intrinsic) small gap material **2**. Space charge regions form; on the left hand side by positively charged ionized dopant atoms; on the right hand side by the increased electron density.
- In other words: In the low gap semiconductor we have carrier **accumulation** [like in Si MOS devices](#).

What happens if we apply a bias U ?

- Well, draw the band diagrams. Move one side up by $e \cdot U$, and [make sure that the band discontinuities do not change](#). What you get for the structure from above is shown below.



The current is always carried by majority carriers (electrons in our case). Inspecting the (exaggerated) drawings, it is clear that this is relatively easy from left to right, but *not* from right to left. Without going into the details of the characteristics, there are several novel features emerging with possible uses for devices. The first one of these effects is clear; the following ones need consideration:

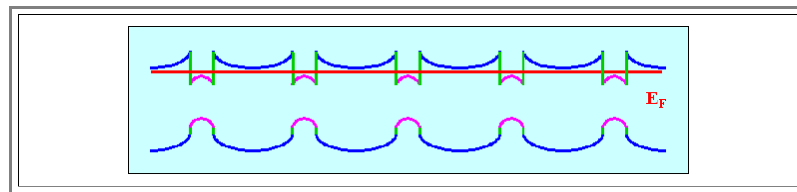
- We may use an isotype heterojunction to **inject** majority carriers from the wide band gap material into the small band gap material [as in the case of diode-type junction](#).
- We may use an isotype heterojunction to **spatially separate** the carriers generated by doping in the wide band gap material from the doped region.
- We might have peculiar **new quantum effects**.

While the first point is relatively clear, including its usability for light emitting devices ([again](#), try to figure this out yourself), the second and third point need some explaining.

Modulation Doping

Let's look at the isotype junction in equilibrium *again* to understand the second point.

- What happened is essentially that the electrons missing in the space charge region on the left hand side were transferred to the potential dip on the right hand side. Of course, electrons are also running down the slope from the right, but the essential contribution is from the wide band gap material on the left (which, after all, is the cause for the dip).
- Since we picked a highly doped material **1** and a lightly doped material **2**, we now have **a lot** of electrons (their density is essentially given by the doping in material **1**) in a crystal with **few** ionized dopant atoms.
- And what this means is that we now have a *high density of highly mobile electrons*, because the mobility at high doping density is always severely decreased by scattering at the ionized dopants – cf. the [paragraph to this topic](#). This effect is most pronounced at low temperatures and can lead to a mobility enhancement of an order of magnitude or even more.
- This is not only generally useful, but can be carried to extremes. All we have to do is to make *sandwiches* as shown below.



- With properly chosen dimensions, deep potential wells will form in the low band gap material that contain most of the electrons from the highly doped wide band gap material. This amounts to a novel way of effectively doping material **2**, called **modulation doping**.

If the potential wells are small enough (which is usually the case), the confinement of the electrons in the wells leads to pronounced quantum effects – we therefore call these potential wells "quantum wells" (**QW**) and distinguish **single quantum wells (SQW)** and **multiple quantum wells (MQW)**.

- An **SQW** is obtained by sandwiching just one small gap semiconductor, a **MQW** as shown above. The [introductory picture](#) of the heterojunction subchapter showed examples of both types.
- We may even improve on that by inserting a very thin layer (**1 . . . 10 nm**, say) of intrinsic material of a suitable band gap between the two basic materials. If properly done, this layer, while not impeding carrier flow into the potential wells for equilibration, keeps the carriers from being scattered at the interface and thus increases the mobility even more.

But with that we have not yet exhausted the possibilities of heterojunctions – we will now turn to special quantum effects.

Quantum Confinement Effects

Let's consider the peculiar quantum effects in modulation doped structures by looking at some typical dimensions.

- The width of the various space charge layers must still be given by formulas not too different from the ones we had for Si. For a **GaAlAs/GaAs** system with a high doping around 10^{18} cm^{-3} in the wide band gap **GaAlAs** side, the width of the dips with the high electron density on the **GaAs** side is about **5 ... 10 nm**, while the lateral extension is large by comparison.
- [The mean free path length](#) of the (highly mobile) electrons is larger than the thickness of the potential dip (better called potential well for the multi-junction configuration shown above) and this means that we now have essentially a *two-dimensional* electron gas.

What does that mean? Especially if we make the thickness of the layers extremely thin?

- It means that we have a periodic crystal in two dimensions (**x** and **y**) and a one-dimensional potential well in the **z**-direction, which is always the direction used in the pictures above.
- The relevant Schrödinger equation is easy to write down, especially in the free electron approximation with a constant potential (= **0**) in **x**- and **y**-, and a potential **V(z)** in **z**-direction:

$$-\frac{\hbar^2}{2} \left(\frac{1}{m_x^*} \cdot \frac{\partial^2}{\partial x^2} + \frac{1}{m_y^*} \cdot \frac{\partial^2}{\partial y^2} + \frac{1}{m_z^*} \cdot \frac{\partial^2}{\partial z^2} \right) \psi(\mathbf{r}) - e \cdot V(\mathbf{z}) \cdot \psi(\mathbf{r}) = E \cdot \psi(\mathbf{r})$$

- This equation is solved by

$$\psi(r) = \psi_{\text{vert}}(z) \cdot \psi_{\text{lateral}}(x,y)$$

- The two functions $\psi_{\text{lateral}}(x,y)$ and $\psi_{\text{vert}}(z)$ are decoupled, the solutions can be obtained separately. For the lateral part we simply have

$$\psi_{\text{lateral}}(x,y) = \text{Solutions of the two-dimensional free electron gas problem}$$

- The vertical part of the solution comes from solving the remaining one-dimensional Schrödinger equation

$$\left(-\frac{\hbar^2}{2m_z^*} \cdot \frac{\partial^2}{\partial z^2} - e \cdot V(z) \right) \psi_{\text{vert}}(z) = E_{\text{vert}} \cdot \psi_{\text{vert}}(z)$$

It is rather clear that the structure of the two-dimensional problem will not be much different from that of the common three-dimensional problem if we introduce a periodic potential in (x,y) . We simply obtain Bloch waves in two dimensions instead of plane waves for $\psi_{\text{lateral}}(x,y)$. The energy eigenvalues are unchanged, too, they [were for the free electron gas](#) (using the [effective masses](#) by now).

$$E_{\text{lateral}} = \frac{\hbar^2 k_{\text{lat}}^2}{2m_{\text{lat}}^*}$$

- The solution of the one-dimensional problem in z -direction depends of course on the precise shape of $V(z)$, but as a general feature of potential wells we must expect a *sequence of discrete energy levels*. For the most simple case of a rectangular well (with infinite height), standard calculations show that

$$E_{\text{vert}} = \frac{(\hbar \pi)^2}{2m_z^*} \cdot \frac{j^2}{d_z^2}$$

- With $j = 1, 2, 3, \dots$ = quantum number, and d_z = thickness of the potential well.

The total energy of an electron is now given by

$$E_{\text{total}} = E_{\text{lat}} + E_{\text{vert}}$$

$$E_{\text{total}} = \frac{(\hbar k_{\text{lat}})^2}{2m_{\text{lat}}^*} + \frac{(\hbar \pi)^2}{2m_z^*} \cdot \frac{j^2}{d_z^2}$$

- This "simply" means that the states in the conduction bands are now a *discrete series* given by the quantum number j with a density of states per level of

$$D_{\text{lat}} = \frac{\hbar \cdot m_{\text{lat}}^*}{2\pi} = \text{constant}$$

- If you like to try your hand at a little math: The formula for D_{lat} is rather easy to obtain if you follow the recipe for the three-dimensional **DOS** for this case.

What do we get from this? Well, a lot of special effects for enthusiastic solid state physicists, but also some (not necessarily big) advantages for devices. *However.....*

- Each **quantum well layer** is now something like a *one-dimensional* atom – in contrast to the three-dimensional real atoms where the wave functions of the electrons were confined in all three directions. If we move these "atoms" close together in the z -direction, there must be a point where the wavefunctions in z -direction (the $\psi_{\text{vert}}(z)$) start to overlap and do all the things *real atoms do at close distance*.

- The energy levels change and split, and – in analogy to a crystal formed by real atoms – a one-dimensional energy band may start to develop with an *energy range that is given by the geometry of the system*, i.e., the thickness of the layers, for a **multi quantum well structure**.

This is a momentous statement! Think about it!

- ▶ It means that we can make materials with energetic properties that we can tailor *at will* (within bounds and limits, of course). We no longer must just live with bandgaps and other properties that mother nature provides, *we now can make our own systems*. At least in principle.
 - Something like that we call a **metamaterial**.
 - Multiple and single quantum wells are already part of recent devices as shown in the [last module](#) of this chapter.