Radiation Equilibrium (in Everything Including Direct Semiconductors)

Advanced

- The analysis of the radiation equilibrium of a "black body" by **Max Planck** in **1900** i.e. exactly **100** years ago, lead to the discovery of quantum mechanics; his law for the radiation density as a function of frequency is seen as the first "quantum" law.
  - Max Planck had a hard time in believing his own result since he introduced the "quantum hypotheses" only as a mathematical tool to play around with, and was surprised by its wild success in explaining facts hitherto not understood.
  - His formal way of deriving at the black-body formula was quite involved. We can do much better, however, because we do not have to discover quantum mechanics - we already know its basic features.
- Essentially, we want to know the distribution of photon energies in a piece of semiconductor with a certain volume $V = L^3$ (for simplicity) which is in thermodynamical equilibrium at some temperature T. Don't confuse this L with the diffusion length !
  - We solved a similar problem already when we looked at the distribution of *electron* energies in a piece of semiconductor with a certain volume $V = L^3$; i.e. when we went through the free electron gas model.
  - There is no reason why we should not follow the free electron gas model. We do not have to solve the Schrödinger equation because we already know that photons are waves described by some $\exp\,(i\underline{k}\cdot\underline{r} - \omega t)$ with $\omega = 2\pi\,\nu$
  - With that, we also know that the boundary conditions imposed by the finite crystal will only allow wave vectors that fit into the crystal and form standing waves.
- All we have to do then is to figure out the *density of states* at the energy **hν**, and the *probability* $f_{ph}(h\nu)$ that these states are occupied .
  - For the density of states we obtain exactly as in the free electron gas

$$\boxed{D_{ph}(k)\cdot d\underline{k} \;=\; \frac{k^2}{\pi^2}\cdot d\underline{k}}$$

  - and it has been taken into account that there are *two* polarization states per photon for each k .
  - Rewriting this for photon energies **hν** using $k = 2\pi\nu\cdot n_{ref}/c$ with n_{ref} = refractive index of the semiconductor, yields

$$\boxed{D_{ph}(h\nu)\cdot d\nu \;=\; \frac{8\cdot\pi\cdot n_{ref}^3\cdot(h\nu)^2}{h^3\cdot c^3}\cdot d(h\nu)}$$

- What is the probability that the states at some energy **hν** are occupied? For electrons - which are Fermions - this was given by the *Fermi-Dirac distribution* which made sure that only *one* electron could occupy a given state.
  - Photons, however, are bosons and *any* number can share a given state. We therefore must resort to the **Bose-Einstein distribution** given by

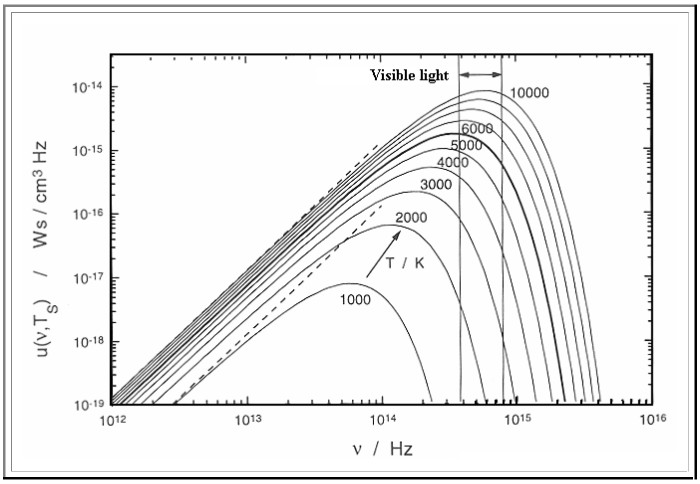
$$\boxed{f_{ph}(h\nu) \;=\; \frac{1}{\exp\,(h\nu/kT) \;-\; 1}}$$

- The *density* $u_\nu\cdot d\nu$ of photons in the frequency interval ν, $\nu + d\nu$ (and, as always, per volume unit) is then given by the product of the density of states and the probability of occupation, it is

$$\boxed{u_\nu d\nu \;=\; \frac{\pi\cdot n_{ref}^3\cdot(h\nu)^2}{h^3\cdot c^3\cdot\exp\,(h\nu/kT)\;-\;1}\cdot d(h\nu)}$$

  - This *is* Plancks formula for the number of photons per volume element in the energy interval **hν**, **h ν + d(hν)** - derived in just a few easy steps!
- If you compare it with some text book formula, you may find a different version - different by a factor **hν**!

  - This is because ususally it is not the *photon density*, but the *energy density* that is considered. The total energy contained in a volume element between , $h\nu + d(h\nu)$ is of course simply the *number* of photons with that energy interval *times* their energy $h\nu$.
  - The energy density, if plotted, gives the well known spectral intensity curves that made Planck famous.



- If we want to know how many photons we have with the *band gap energy*, we only have to insert $h\nu = E_g$ to get the answer.
  - But how about $h\nu = E_g/2$ or any other energy inside the band gap? After all, photons with these energies can not be created in the semiconductor, while they have a certain density according to Plancks formula.
  - Well, as in the free electron gas model (which does not have band gaps after all) we have made approximations that do not quite apply to the case of semiconductors. We have assumed a *black body* that absorbs and emits at *all* frequencies - and this is *not true* for a "cool" semiconductor.
  - Essentially, we should enter the *proper* density of states for photons, but this is far beyond the scope of the course.
- On the other hand, we can make a detailed inspection of the the thermodynamic equilibrium *just for the frequencies corresponding to the band gap*.
  - This was what *Einstein* contributed to *this* field (should have been his 3rd or 4th Nobelprize). It will yield expressions for the "*Einstein coefficents*" crucial for Lasers and is demonstrated in another advanced module

- Finally, a few words to the reason *why* Plancks radiation law was so seminal and what went wrong before it was found.
  - Before Planck, others had considered radiation equilibrium - essentially people wanted to know why *all* hot bodies "glowed" pretty much the same way, independent of their composition.
  - Based on a the "final" knowledge of electromagnetism as put down by **Maxwell** and a fully developed (classical) theory of thermodynamics, **Raleigh** and **Jeans** showed - beyond a trace of doubt - that the energy density $h\nu \cdot u_\nu \cdot d\nu = E_\nu \cdot d\nu$ of electromagnetic radiation coming off a hot body must be

$$\boxed{E_\nu \cdot d\nu = \frac{8 \cdot \pi \cdot n_{ref}^3 \cdot (h\nu)^2}{h^3 \cdot c^3 \cdot kT} \cdot d(h\nu)}$$

  - Big problem! The energy density increases forever, and there should be tremendous amounts of **UV** and **X**-rays coming out of a hot body. This is obviously not true, the term "**ultraviolet catastrophe**" was coined.
- But Raleigh and Jeans made no mistake - something was *fundamentally* wrong with classical physics as a discipline. The ultraviolet catastrophe, in fact, was one of the many stumbling boocks of classical physics at the close of the **19th** century. Let's see what went wrong:
  - Essentially , Raleigh and Jeans' formula says:

$$\boxed{\text{Energy density } E_\nu \cdot d\nu = \text{density of states } D(E) \cdot kT}$$

  - This is nothing but the **equipartition theorem** that states that in a given system every energetic degree of freedom is imbued with the *same* average energy kT . Since the energy fluctuations - with kT as the average value - can have *any* value (just with different probabilities), *all* energies (= energy levels in modern parlor), including the very large ones - can be reached and populated.

- The correct formula of Planck now essentially states that
  - Energy density $\mathbf{E_\nu d\nu}$ **=** density of states ***D*(*E*)** times $\mathbf{h\nu}$ times *distribution function*.
  - Energy now comes in quanta $\mathbf{h\nu}$, and for finite temperatures there might not be any to populate the high-up states.
- Big difference! But not for low energies, where the Raleigh-Jeans law is a good approximation of Plancks law.