Solution to Exercise 3.1.1

Illustration

- How far off from perfection is a **1000 Ωcm** Si crystal (at **300 K**)? The resisitvity given is about the best (i.e. highest) value that **Si** crystal growers can achieve on a routine base.
  - Consider what level of dopants corresponds to **1000 Ωcm** ? How far away from perfection (= truly intrinsic behavior) are the crystal growers in terms of dopant concentration?
  - For that you must know the *intrinsic* carrier density and resistivity at room temperature in **Ωcm**. Calculate the carrier density with the numbers and relations provided and find some suitable value for the mobility (from the various illustrations in chapter 2).
- This exercise not only demands that we generate numbers from some general formulas (which is not as easy as it looks), but also gives an idea of how close we can get to the real numbers with our simple models.

- First some numbers from the literature. According to "Semiconductor Materials", the intrinsic electrical conductivity of **Si** at **300 K** is
3.16 µS/cm. The NSM archive has rather similar numbers.
  - **[S] =** "Siemens" is a quaint German measure of conductivity, it is simply **1/Ω**
  - This translates into a room temperature resistivity ρ of $\rho = 1/\sigma = 316\,000\ \Omega\text{cm}$.
  - Alternatively , numbers for the intrinsic carrier density found in the sources given above or in arbitrary books are somewhere in between $1.00 \cdot 10^{10}\ \text{cm}^{-3}$ or $1.38 \cdot 10^{10}\ \text{cm}^{-3}$.
- Lets see if we can get numbers like this *by calculation*:
  - The carrier density is given by

$$n^e = N^e_{eff} \cdot \exp - \frac{E_C - E_F}{kT}$$

  - N^e_{eff} can be estimated from the free electron gas model in a fair approximation to

$$N^e_{eff} = 2 \cdot \left(\frac{2\,\pi\, m\, kT}{h^2}\right)^{3/2}$$

  - The dimension of this N^e_{eff} is

$$[N_{eff}] = kg^{3/2} \cdot eV^{3/2} \cdot eV^{-3} \cdot s^{-3} = kg^{3/2} \cdot eV^{-3/2} \cdot s^{-3}$$

  - That is a bit strange. Nevertheless it is right - try to do something about the **kg**! If you have problems of figuring out how to get the proper dimension m^{-3}, use the link.
- Inserting numbers ($m_e = 9{,}109 \cdot 10^{-31}$ kg; $k\cdot T = 1/40$ eV, $h^2 = (4{,}1356 \cdot 10^{-18})^2\ eV^2s^2 = 1{,}71 \cdot 10^{-35}\ eV^2s^2$), we obtain

$$\begin{aligned} N_{eff} &= 4.59 \cdot 10^{15} \cdot T^{3/2}\ \text{cm}^{-3} \\ &= 2.384 \cdot 10^{19}\ \text{cm}^{-3} \qquad T = 300\ \text{K} \\ &= 2.384 \cdot 10^{25}\ \text{m}^{-3} \end{aligned}$$

  - The intrinsic carrier density thus is

$$n^e = 3.22 \cdot 10^{19}\ \text{cm}^{-3} \cdot \exp - \frac{E_C - E_F}{kT} = 3.22 \cdot 10^{19}\ \text{cm}^{-3} \cdot \exp - \frac{0.55\ \text{eV}}{0.025\ \text{eV}} = 9 \cdot 10^{9}\text{cm}^{-3}$$

  - This is just a little bit smaller than than the values given above; rather amazing, considering that the free electron gas model is just a very simple approximation.

- Now we can see what kind of mobility μ we would get with $n_i = 1 \cdot 10^{10}$ cm^{-3} and a conductivity $\sigma = 3.16\ \mu S/cm = 3.16 \cdot 10^{-6}\ \Omega^{-1} cm^{-1}$
  - We had the simple law $\sigma = 2e\mu n_i$ (the factor two takes into account that we have holes and electrons), and thus $\mu = \sigma / 2e\, n_i$. This gives us

$$\mu = \frac{3.16 \cdot 10^{-6}}{2 \cdot 1{,}602 \cdot 10^{-19} \cdot 1 \cdot 10^{10}}\ \Omega^{-1} \cdot cm^{-1} \cdot C^{-1} \cdot cm^3$$

  - With $[\Omega] = [V/A] = [V \cdot s/C]$ we have

$$\mu = 986\ cm^2 \cdot s^{-1} \cdot V^{-1}$$

  - as an expected result. The unit $[cm^2 \cdot V^{-1} \cdot s^{-1}]$ comes from the original definition of **μ**, which was (drift) velocity divided by field strength.
- Looking around a bit we find tabulated values of, e.g., **1400 cm²/Vs**, which is just off by a factor of two - and that we do not take seriously. So, what have we learned so far?
  - **1.** It is not so easy to really *calculate* the intrinsic properties. Getting the right order of magnitudes is already pretty good. This is due, of course, to the fact that we have approximations a plenty, coupled with lots of exponentials which are quite sensitive to small changes in the argument.
  - **2.** If we accept an intrinsic carrier concentration for one kind of carrier at room temperature around $n_i = 1 \cdot 10^{10}$ cm^{-3}, we would need a dopant concentration that is at least an order of magnitude smaller if we want to claim truly intrinsic properties. That means we demand

$$N_{dop} \leq 1 \cdot 10^{-9}\ cm^{-3} \leq 20\ ppqt$$

  - Find out what **ppqt** means yourself.
  - The minimum doping concentration N_{min} achievable (corresponding to the maximum *resistivity* ρ_{max} of **1000 Ωcm** or the minimum *conductivity* σ_{min} of $1 \cdot 10^{-3}\ \Omega^{-1} \cdot cm^{-1}$) must be about

$$N_{min} = \frac{316\ 000}{1000} \approx 300\ n_i = 3 \cdot 10^{12}\ cm^{-3}$$

  - In the "master" curve for resistivity vs. doping, we find a value between $5 \cdot 10^{12}$ cm^{-3} and $1 \cdot 10^{13}$ cm^{-3}, so again we are close enough.
- The final answer thus is: We are still at least a factor of **100** away from "perfection" with respect to unwanted doping.
  - And of course, we can not make any statement about the perfection achieved with respect to impurities that do not influence the carrier concentrations