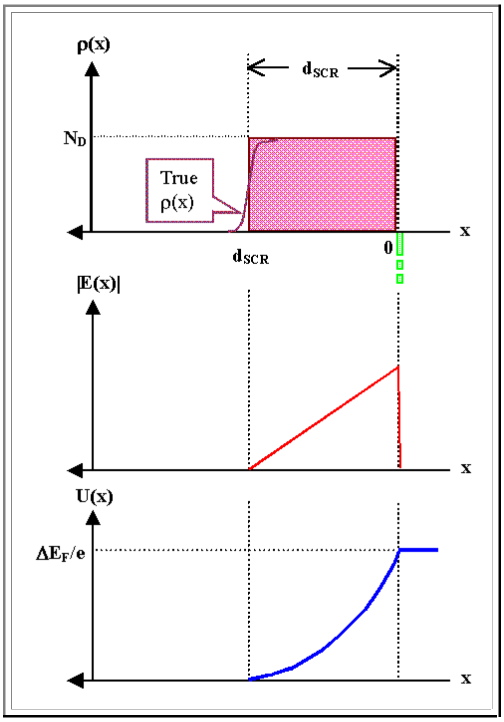
Space Charge Region and Poisson Equation

Illustration

- We start from a (constant) distribution of positive charges (for **n**-doped semiconductors) in the space charge region.
  - The corresponding negative charges are all on the surface; the charge distribution is shown in the first frame of the illustration.
  - Poisson's equation states that (for the one-dimensional case).

$$\epsilon\,\epsilon_0 \frac{d^2 V(x)}{dx^2} = -\rho(x) = e \cdot N_D$$

  - For $0 < x < d_{SCR}$ and $= 0$ everywhere else. (We can also use the voltage $U(x)$ instead of $V(x)$ if we think as $V(x = \infty) = 0$). That will also be reflected in the choice of boundary conditions made below.
- The drawing below shows the situation, including the slight approximation implicit in our choice of $\rho(x)$. Note that the x -direction ist to the left in this case.



  - The first straight-forward integration yields $dU/d(x)$ which is the electrical field strength $E_x = -dU/dx$, or

$$\epsilon\epsilon_0 \frac{dV(x)}{dx} = -\epsilon\,\epsilon_0 E_x = e \cdot N_D \cdot x + \text{const.}$$

  - With the boundary condition $E_x(x = d_{SCR}) = 0$, we obtain (always for the interval $x = 0$ and $x = d_{SCR}$, of course):

$$e \cdot N_D \cdot d_{SCR} + \text{const} = 0$$

$$\text{const} = -e \cdot N_D \cdot d_{SCR}$$

$$E_x = \frac{1}{\epsilon\epsilon_0} \cdot (e \cdot N_D d_{SCR} - e \cdot N_D \cdot x)$$

  - The second integration yields

$$\epsilon\epsilon_0 \cdot U(x) = \frac{e \cdot N_D \cdot x^2}{2} - e \cdot N_D \cdot d_{SCR} \cdot x + \text{const.}$$

- With the boundary condition **U(d_{SCR}) = 0**, we obtain .

$$-\frac{e \cdot N_D \cdot d^2_{SCR}}{2} + \text{const.} = 0$$

$$\text{const.} = \frac{e \cdot N_D \cdot d^2_{SCR}}{2}$$

- Using the proper expression for the integration constant gives gives us the complete voltage function or the shape of the band bending

$$\epsilon\epsilon_0 \cdot U(x) = \frac{e \cdot N_D \cdot x^2}{2} - e \cdot N_D \cdot d_{SCR} \cdot x + \frac{e \cdot N_D \cdot d^2_{SCR}}{2}$$

- The width of the space charge region can be obtained by considering the voltage at $x = 0$, where we have $U(x = 0) = \Delta E_F/e$.Using this we obtain

$$\frac{\epsilon\epsilon_0}{e} \cdot \Delta E_F = \frac{e \cdot N_D \cdot d^2_{SCR}}{2}$$

This gives us the final result for the width of the space charge region

$$d_{SCR} = \frac{1}{e} \cdot \left(\frac{2\Delta E_F \cdot \epsilon\,\epsilon_0}{N_D}\right)^{1/2}$$

The corresponding curves are shown in the drawing above. We obtained the same formula as before, but now we have a better awareness of the approximations it contains.

- The positive charge distribution was assumed to be box-shaped and uniform. This is a rather good approximation; the drawing indicates the precise shape of the charge distribution for comparison.
- The counter charges are described by a δ -function at the surface; these charges only enter the calculation in the indirect form of a boundary condition.