Solution to Exercise 2.3.5-1

Illustration

We want to show that the following two equations are equivalent for *equilibrium*:

$$n_e^p(U)\Big|_{\substack{\text{SCR}\\\text{edge}}} = n_e^n(U)\Big|_{\substack{\text{SCR}\\\text{edge}}} \cdot \exp -\frac{e(V^n + U)}{kT}$$

$$n_e^p(U=0) = \frac{n_i^2}{n_h^p(U=0)}$$

- The first equation then simplifies to

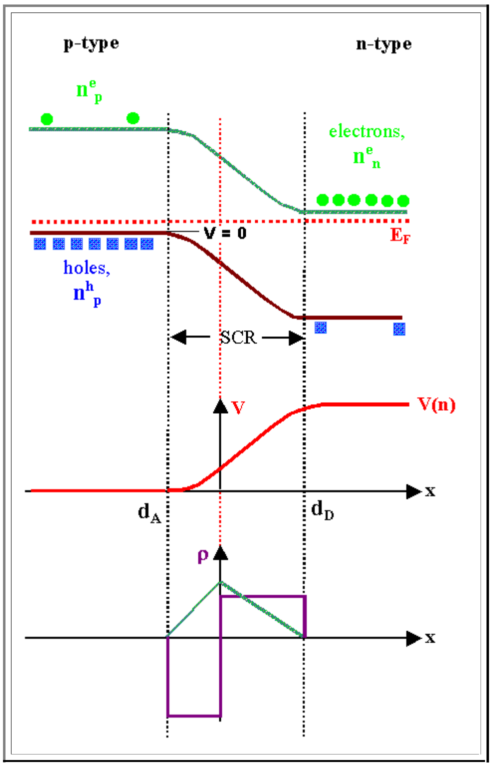
$$n_e^p(U)\Big|_{\substack{\text{SCR}\\\text{edge}}} = n_e^n(U)\Big|_{\substack{\text{SCR}\\\text{edge}}} \cdot \exp -\frac{eV^n}{kT} = n_e^n(U)\Big|_{\substack{\text{SCR}\\\text{edge}}} \cdot \exp -\frac{\Delta E_F}{kT}$$

Start with the equation for the majority carrier concentration $n_h^p(U=0)$ in general and the definitions of the energies:

$$n_h^p(U=0) = N_{eff}^{\ p} \cdot \exp -\frac{E_F - E_V^{\ p}}{kT}$$

$$e\cdot V^n = \text{Difference of band edges} = E_V^p - E_V^n = E_C^p - E_C^n = \Delta E_F$$

- Consult the solution to the Poisson equation if you are unsure (the relevant diagram is reprinted below) and recall that in the band diagram, the energy scale refers to electrons, which carry a negative electric charge – so that the electrostatic potential contributes with a negative sign.
- Also note that E_F , of course, is constant in equilibrium, and ΔE_F thus refers to the difference in Fermi energies *before the contact* !



- E_V^p thus can be expressed as $E_V^p = E_V^n + \Delta E_F$.
  - This brings you already to the **n**-side. However, you want to find n_e^n in the equation, and for that you need a factor $E_C^n - E_F$.
  - So, express E_V^n in terms of E_C^n via $E_V^n = E_C^n - E_g$ with E_g = band gap. This yields

$$\boxed{n_h^p(U=0) \;=\; N_{eff}^{\,p} \cdot \exp - \frac{E_F - E_C^n + E_g - \Delta E_F}{kT}}$$

- You now have terms that occur in the definition of the electron density in **n-Si** [namely, $E_F - E_C^n = -(E_C^n - E_F)$] and for the intrinsic carrier density (namely, E_g).
  - So, multiply with N_{eff}^n / N_{eff}^n, remember that $n_i^2 = N_{eff}^p \cdot N_{eff}^n \cdot \exp - E_g/(kT)$, and $1/n_e^n = 1/N_{eff}^n \cdot \exp[(E_C^n - E_F)/(kT)]$; thus, you have

$$\boxed{n_h^p(U=0) \;=\; \frac{n_i^2}{n_e^n} \cdot \exp \frac{\Delta E_F}{kT}}$$

- This gives for n_e^n:

$$\boxed{n_e^n(U=0) \;=\; \frac{n_i^2}{n_h^p} \cdot \exp \frac{\Delta E_F}{kT}}$$

  - We now can substitute n_e^n in our first equation and obtain

$$\boxed{\begin{aligned} n_e^p \Big|_{\substack{\text{SCR}\\\text{edge}}} &= \frac{n_i^2}{n_h^p} \cdot \exp \frac{\Delta E_F}{kT} \cdot \exp - \frac{\Delta E_F}{kT} \\ \Rightarrow \quad n_e^p &= \frac{n_i^2}{n_h^p} \end{aligned}}$$

- That is exactly the second equation – **Q.E.D.**