Solution to Exercise 2.1-2: Density of States for Lower Dimensions

Illustration

- Calculate the density of states for a one-dimensional semiconductor ("quantum wire") and for the two-dimensional case.
  - Draw some conclusions from the results.
- The number of states ***Z*(*k*)** up to a wave vector ***k*** is generally given by

$$Z(k) \;=\; \frac{\text{Volume of "sphere" in } m \text{ dimensions}}{\text{Volume of state}}$$

  - The volume V_m of a "sphere" with radius ***k*** in ***m*** dimension is

$$V_m(k) \;=\; \begin{cases} \text{Volume of sphere} = 4/3\,\pi \cdot k^3 & \text{for } m = 3 \\ \text{Volume = area of circle} = \pi \cdot k^2 & \text{for } m = 2 \\ \text{Volume = length} = 2k & \text{for } m = 1 \end{cases}$$

- The density of states ***D*(*E*)** follows by substituting the variable ***k*** by ***E*** via the dispersion relation ***E*(*k*)** and by differentiation with respect to ***E***.
  - One obtains the following relations:

$$D_m(E) \;\propto\; \begin{cases} (E)^{1/2} & \text{for } m = 3 \\ \text{const.} & \text{for } m = 2 \\ (E)^{-1/2} & \text{for } m = 1 \end{cases}$$

- The consequences can be pretty dramatic. Consider, e.g. the concentration of electrons you can get in the three case for $E \approx 0$ **eV**, i.e close to the band edge.
- The question, of course, is: Are there **1**-dim. and **2**-dim. semiconductors? The answer is: yes – as soon as the other dimensions are small enough, we will encounter these cases. We will run across examples later in the lecture course.