Phase and Group Velocity

Basics

- The velocity **v** of the electron in the free electron gas model is the *velocity with which the phase of the wave moves*, it is called **phase velocity** $\mathbf{v_P}$.
  - As long as a single plane wave symbolizes all that is to a particle, the *phase* velocity is automatically also the *particle* velocity **v**.
- This is no longer true, however, for particles that are described as a *superposition* of (plane or other) waves with all kinds of wave vectors - and that means most *real* particles.
  - While each individual wave of the set that now describes the particle still travels with its individual phase velocity $\mathbf{v^i_P}$, the *maximum of the total amplitude of all waves* - signifying the most likely place to find the particle - travels with a velocity that must be identified with the particle velocity **v** and that is called **group velocity**.
  - That the *group velocity* may be totally different from the *phase velocity* is nicely demonstrated by the example of standing waves, obtained by just combining two plane waves with wave vectors $\underline{\boldsymbol{k}}$ and $-\underline{\boldsymbol{k}}$. These waves have the same magnitude of the phase velocity, just opposite signs.
- The result is a *standing wave* with maxima and minima that are fixed in space; we have group velocity $\mathbf{v_G = 0}$. We may not really know where the particle is (locating it makes no sense under these coditions) but it certainly isn't going anywhere!
- The group velocity for a particle still characterized by *one* wave vector ***k*** as. e.g. in a Bloch wave, is calculated by the simple formula

$$v_G = \frac{1}{\hbar} \cdot \frac{dE(k)}{dk}$$

  - i.e. it follows directly from the dispersion relation $E = E(k)$.
- This distinction is essential to avoid confusion between the over-simplified quantum mechanical picture of the free electron gas, and the treatment of electrons (and holes) as particles later on.
  - While the electron in the *free electron gas* would move through the lattice with a rather large velocity because group and phase velocity are identical, the "real" electrons are rather sluggish.
  - This is of course a direct consequence of the fact that only electrons and holes near the band edges are important where the group velocities are low because $E(k) \approx$ **constant**.
  - *In other words:* The electrons we are concerned with resemble much more standing waves than fast moving plane waves. Or, in yet other words: They spent a lot of time hanging around the atoms before they move on to some other atom.