2.3.3 Shockley-Read-Hall Recombination

- In this chapter we take a closer look at the generation and recombination of carriers. Even the simple treatments given so far – cf. the formulas for the **p-n** junction – made it clear that *generation and recombination are the major parameters that govern device characteristics and performance*.
  - First, we will treat in more detail the band-to-band recombination in *direct semiconductors*, next the recombination via defects in indirect semiconductors, and for this we introduce and use the "**Shockley -Read- Hall Recombination**" or **SRH** model.
  - However, we will just sort of scratch the subject. In an advanced module some finer points to recombination are treated; here we will stick to fundamentals.
- First a few basic remarks. Generally, we do not only have to maintain energy and momentum conservation for any generation/recombination process, we also have to assure that we keep the minimum of the free enthalpy, or in other words, we have also to consider the **entropy** of these processes. These requirements transform into the conditions
- **1.** $\underline{k} - \underline{k}' = \underline{g}$ as an expression of the (crystal) *momentum conservation*.
- **2.** $E^{e} - E^{h} = \Delta E^{\text{something else}}$ for energy conservation.
  - We have $E_C - E_V = \Delta E^{\text{something else}}$ because the electrons and holes recombining are always close to the band edges for *energy conservation*.
  - $\Delta E^{\text{something else}}$ refers to the unavoidable condition that "something else" has to *provide* the energy needed for generation, or must *take away* the energy released during recombination.
- **3.** Now we look at the *entropy*. Recombination *reduces* the entropy of the system (empty bands are more orderly than bands with a few wildly moving holes and electrons). The "something else" that takes energy out of the system may in addition take some entropy out of it, too. However, no easy law can be formulated.
- The first two points determine if a recombination/generation event – which we from now on are going to call an **R/G**-event – can take place *at all*, i.e. if it is allowed; the third point comes in – in principle – when we discuss the *probability* of an allowed **R/G**-event to take place. This insight, however, will only be used in an indirect way in what follows.
- The major quantities are the **recombination rate** R and the **generation rate** G.
  - The recombination rate R is the more important one of the two. It is related to the carrier density $n_{e,h}$ by

$$\frac{dn_{e,h}}{dt} = -R$$

  - It is *always* directly given by the rate at which the carrier density *decreases* (the minus sign thus makes R a positive quantity) and it does not matter which carrier type we are looking at because $dn_e/dt = dn_h/dt$ as long as the carriers disappear in pairs by recombination.
  - Note that the equilibrium condition of constant carrier density does *not* mean that there is no dynamics anymore in the charge carrier population (i.e. that the carriers remain where they are): When n remains constant, this just means that as many carriers recombine as are generated, since n is an average quantity. (That this is not unlike the drift velocity of electrons which can be zero despite large thermal velocities of the individual electrons.)

Recombination and Generation in Direct Semiconductors

- If we first look at *recombination in direct semiconductors*, we need holes and electrons at the *same position in the band diagram*; i.e. in *$\underline{k}$-space*. However, that does not imply that they are at the *same position in real space*. For a recombination event they have to find each other; i.e., we also need them to be at about the same position in real space.
  - The recombination rate R thus must be proportional to the two densities, n_e and n_h, because the probability of finding a partner scales with the carrier density. We thus can write down the recombination rate R as

$$R = r \cdot n_e \cdot n_h = r \cdot N_{\text{eff}}^{h} \cdot N_{\text{eff}}^{e} \cdot \exp - \frac{E_C - E_F{}^{e}}{kT} \cdot \exp - \frac{E_F{}^{h} - E_V}{kT}$$

  - With r = proportionality constant, having the dimensions of volume/time; we will come back to this later. We also only assume only *local* equilibrium as evidenced by the use of quasi Fermi energies.
- We can rewrite this equation as follows

$$R = r \cdot N_{eff}^{h} \cdot N_{eff}^{e} \cdot \exp \frac{-E_C + E_F^{e} - E_F^{h} + E_V}{kT} = r \cdot N_{eff}^{h} \cdot N_{eff}^{e} \cdot \exp - \frac{E_C - E_V}{kT} \cdot \exp - \frac{E_F^{h} - E_F^{e}}{kT}$$

- Using our old relation for the intrinsic carrier density n_i

$$n_i^{2} = N_{eff}^{h} \cdot N_{eff}^{e} \cdot \exp - \frac{E_C - E_V}{kT}$$

- we finally obtain

$$R = r \cdot n_i^2 \cdot \exp \frac{E_F^{e} - E_F^{h}}{kT}$$

- Note again that we have not invoked *total* equilibrium, but only *local* equilibrium in the bands – we use the quasi Fermi energies $E_F^{e,h}$. That is essential; after all it is recombination and generation that restore equilibrium between the bands and the **SRH** theory only makes sense for non-equilibrium.

If we were to consider *total* thermal equilibrium, we know that the generation rate G must be identical to the recombination rate R; both quasi Fermi energies are identical (= E_F) and $R = r \cdot n_i^2$ applies.

- Note that we did *not* assume intrinsic conditions; the Fermi energy can have any value, i.e. the semiconductor may be doped.
- In essence, we see the following:

The recombination rate in non-equilibrium depends very much on the actual carrier density!

So far it was easy and straigth-forward. Now comes an important point.

- In *contrast to the recombination rate* R, the **generation rate** G does *not* depend (very much) on the carrier density; it is just a reflection on the thermal energy contained in the system and therefore *pretty much constant*. In other words, under most conditions we have

$$G = G_{therm} \approx \text{constant} \neq R(n)$$

- We may, from the above consideration, equate G under *all* conditions with the recombination rate for equilibrium, i.e.

$$G = G_{therm} = r \cdot n_i^2$$

In non-equilibrium, which will be the normal case for devices under operation, the *difference* $(R - G)$ is no longer zero, but has some value

$$U = R - G$$

- Since R is mostly (but not always) larger than G under non-equilibrium conditions, U is the *net* rate of recombination (or, on special occasions, the *net* generation rate).
- Using the expressions derived so far, we obtain

$$U = R - G_{therm} = r \cdot (n_e \cdot n_h - n_i^2) = r \cdot n_i^2 \cdot \left(\exp \frac{E_F^{e} - E_F^{h}}{kT} - 1 \right)$$

- This equation tells us, for example, how fast a non-equilibrium carrier density will decay, i.e. how fast full equilibrium will be reached, or, if we keep the non-equilibrium density fixed for some reason, what kind of **recombination current** we must expect.

- This is so because U, the difference between recombination and generation, *times* the charge is nothing but a net *current flowing from the conduction band to the valence band* (for positive U).

Determining the Proportionality Constant *r*

- We still need to determine the proportionality constant r.
  - This is not so easy, but we can make a few steps in the right direction. We assume in a purely classical way that an electron (or hole) moves with some average velocity **v** through the lattice, and *whenever it encounters a hole (or electron), it recombines*.
  - The problem is the word "*encounters* ". If the particles were to be small spheres with a diameter d_p, "encountering" would mean that parts of such a sphere would be found in the cylinder with diameter d_p formed by another moving sphere because that would cause a physical contact.
  - Our particles are not spheres, but for the purpose of scattering theory we treat them as such, except that the diameter of the cylinder that characterizes its "scattering size" is called **scattering cross section** σ and has a numerical value that need *not* be identical to the particle size.
- *One* electron now covers a volume **v** · σ per second and *all* N_e electrons of the whole sample (a number, not a density) probe per second the volume

$$V_{probed} = N_e \cdot \mathrm{v} \cdot \sigma$$

- Any time an electron encounters a hole in the volume it probes, it recombines. The absolute recombination rate R_{abs} then is simply the number of encounters per second, occurring in the whole sample.
  - How many holes are "hit" per second? In other words, how many are to be found in the volume probed? That is easy: The *number* N_h of holes encountered in the volume probed by electrons, and thus the recombination rate is

$$N_h = n_h \cdot V_{probed} = n_h \cdot N_e \cdot \mathrm{v} \cdot \sigma = R_{abs}$$

  - Here, n_h is simply the *density* of holes in the sample. You many wonder if this is correct, considering that the holes move around, too, but simply realize that the density of holes is nevertheless constant.
- The formula is a bit unsatisfying, because it contains the volume *density* of holes, but the *absolute* number of electrons.
  - That is easily remedied, however, if we express N_e, the *number* of electrons, by their *density* n_e via $N_e = n_e \cdot V$ with V = sample volume. Using the latter for normalizing the absolute recombination rate to the sample volume, this gives us

$$R = \frac{R_{abs}}{V} = \frac{\text{Recombinations per}}{\text{s and cm}^3} = n_e \cdot n_h \cdot \mathrm{v} \cdot \sigma$$

  - In other words, if we use the *density* of the electron and holes, we obtain a recombination rate *density* , i.e. recombination events per **s** and per **cm**^{3} – as it should be. *As always, we are going to be a bit sloppy about keeping densities and numbers apart. But there is no real problem: Just look at the dimensions you get, and you know what it is.*
- A comparison with the formula from above yields

$$r = \mathrm{v} \cdot \sigma$$

- This leaves us with finding the proper value for σ. Whereas this is difficult (in fact, the equation above is more useful for determining σ from measurements of R than to calculate r), we are still much better off than with r alone:
  - Whereas we had *no idea* about a rough value for r, we do know something about **v** (it is the group velocity of the carriers considered), and we know at least the rough order of magnitude for σ: We would expect it to be in the general range of atomic dimensions (give or take an order of magnitude).
- You might wonder now why we assume that any "meeting" of the elctrons and holes leads to recombination, given that we have to preserve momentum, too. You are right, but remember:
  - We are treating *direct* semiconductors here! Since we only consider the mobile electrons and holes, we only consider the ones at the band edges – and those have the same **k**-vector in the reduced band diagram!

Useful Approximations and the Lifetime τ

We now consider non-equilibrium, but describe it in terms of deviations from equilibrium. Then it is sensible to rewrite the carrier densities (or numbers, *take whatever you like*) in terms of the equilibrium density $n_{e,h}$**(equ)** *plus/minus some delta*:

$$n_{e,h} = n_{e,h}(\text{equ}) + \Delta n_{e,h}$$

- This is one of the decisive "tricks" to get on with the basic equations, because it permits to specify particular cases (e.g. $\Delta n_{e,h} << n_{e,h}(\text{equ})$ or whatever), and then resort to approximations. We will encounter this "trick" fairly often.

We obtain after some shuffling of the terms form the equation for the net recombination rate U

$$U = v \cdot \sigma \cdot \Big(\{n_e(\text{equ}) + \Delta n_e\} \cdot \{n_h(\text{equ}) + \Delta n_h\} - n_e(\text{equ}) \cdot n_h(\text{equ}) \Big)$$

So far everything is still correct. But now we consider a first *special*, but still rather general case:

- We assume that $\Delta n_e = \Delta n_h = \Delta n$, i.e. that only additional electron–hole *pairs* were created in non-equilibrium. We then may simplify the equation to

$$U = r \cdot \Big(\Delta n \cdot \{n_e(\text{equ}) + n_h(\text{equ})\} + \Delta n^2 \Big)$$

- We can simplify even more. For the *extrinsic* case where *one* carrier density – let's say for example n_h – is far larger than n_e or Δn (i.e. we have a **p**-doped semiconductor), we may neglect the terms $\Delta n \cdot n_e(\text{equ})$ and Δn^2 and obtain

$$U \approx r \cdot n_h \cdot \Delta n$$

U was the difference between the recombination and the generation rate. We are now looking at an approximation where only some Δn in the density of the minority carriers is noticeably different from equilibrium conditions (where we always have $U = 0$).

- We thus may write

$$U = R(\text{equ}) + R(\Delta) - G(\text{equ}) = R(\Delta)$$

- Here, $R(\Delta)$ denotes the *additional* recombination due to the **excess minorities**. Remembering the basic definition of R we see that now we have

$$\frac{d(\Delta n_e)}{dt} = -U = -r \cdot n_h \cdot \Delta n_e = -v \cdot \sigma \cdot n_h \cdot \Delta n_e$$

This is a differential equation for $\Delta n_e(t)$, it has the simple solution

$$\Delta n_e(t) = \Delta n_e(t = 0) \cdot \exp - \frac{t}{\tau}$$

- The quantity demanded by the general solution is, of course, the **life time** of the minority carriers. We now have a formula for this prime parameter, it comes out to be

$$\tau = \frac{1}{v \cdot \sigma \cdot n_h} = \frac{1}{v \cdot \sigma \cdot n_{maj}}$$

- The last equality generalizes for both types of carriers – it is always the density of the *majority* carriers that determine the lifetime of the *minority* carriers. This is clear enough considering the "hit and recombine" scenario that we postulated at the beginning

Substituting $r \cdot n_h$ with $1/\tau$ in the equation for U yields

$$U = \frac{\Delta n}{\tau}$$

- In other words: The recombination rate in excess of the recombination rate in equilibrium is simply given by the excess density of minority carriers divided by their life time.

In yet other words:

- The *net current* flowing from the band containing the minority carriers to the other band is given by U (times the elementary charge, of course, and times the total sample volume), because U gives the net amount of carriers "flowing" from here to there! And that is the definition of a current!

This result not only justifies our earlier approach, it gives us the *minority carrier life time* in more basic quantities including (at least parts) of its temperature dependence via the thermal velocity **v** and the majority carrier density n_h – the T-dependence of which we already know.

- Since $1/n_h$ is more or less proportional to the resistivity, we expect τ to increase linearly with the resistivity which it does as illustrated before, at least for resistivities that are not too low.
- A rough order of magnitude estimate gives indeed a good value for many *direct* semiconductors:

$$\sigma \approx 10^{-15}\ \text{cm}^2$$
$$v = 10^{7}\ \text{cm/s} \quad \Rightarrow \quad \tau \approx 10^{-9}\ \text{s} = 1\ \text{ns}$$
$$n_h = 10^{17}\ \text{cm}^{-3}$$

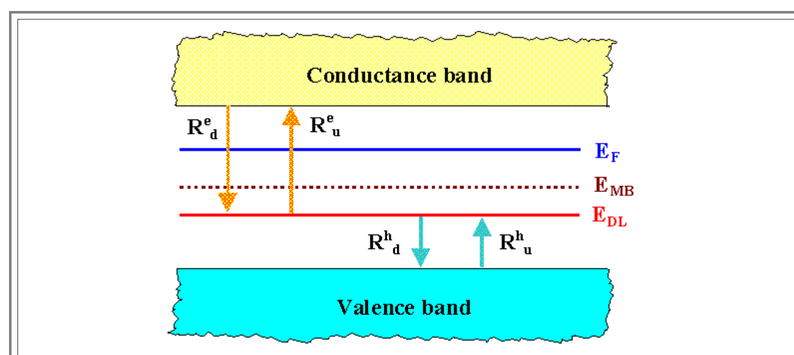
Recombination and Generation in Indirect Semiconductors

In *indirect* semiconductors, *direct* recombination is theoretically impossible or, being more realistic, very improbable.

- In general, a recombination event needs a *third partner* to provide conservation of energy and crystal momentum.
- Under most (but not all) circumstances, this third partner is a *lattice defect*, most commonly an impurity atom, with an energy state "*deep*" in the band gap, i.e. not close to the band edges.
- Recombination then is determined by these "**deep states**" or **deep levels**, and is no longer an intrinsic or just doping dependent property.

How the recombination and generation depends on the *properties of deep levels* is the subject of the proper Shockley–Read–Hall theory (what we did so far was just a warming-up exercise). It is a lengthy theory with long formulas; here we will just give an outline of the important results. More topics will be covered in an even more advanced module.

- First we look at the situation in a band diagram that shows the relevant energy levels plus the mid-band energy level E_{MB}, which will come in handy later on.



- Besides the energy level of the "deep level" defect, we now need *four* **transition rates** instead of just *one* recombination rate:
  - R^e_d, the rate with which *electrons* from the conductance band transit from the conduction band to the deep level, or more simply put, *occupy* the deep level with the energy E_{DL} – in short: the rate with which they are going *down* to the deep level.
  - R^e_u , the rate with which *electrons* occupying the deep level state go *up* to the conduction band.
  - R^h_u, the rate with which *holes* from the valence band go *up* to the deep level – or better: the *electrons* in the deep level go *down* to the valence band, and finally
  - R^h_d, the rate with which *holes* from the deep level go *down* to the valence band – or better: *electrons up* to the deep level.

The equilibrium density of electrons (and holes) on the deep level is, as always, given by the Fermi distribution. We have

- $n^-_{DL} = N_{DL} \cdot f(E_{DL}, T)$ = density of negatively charged deep levels with *one* electron sitting on it, and
- $n^0_{DL} = N_{DL} \cdot [1 - f(E_{DL}, T)]$ = density of deep levels with *no* electron sitting on it. N_{DL} , of course, is the *density* of deep level states, e.g. the density of impurity atoms. It's written with capital N (otherwise used for absolute numbers) to avoid confusion with the carrier densities.
- To make life easier, we assumed that the deep level is normally neutral, i.e. does not contain an unalterable fixed charge, and it can only accommodate one additional electron.

We may now write down formulas for the transition rates in direct analogy to the consideration of the recombination rate in direct semiconductors as given above. For R^e_d we have

$$R^e_d = r \cdot n_e \cdot n^0_{DL} = v \cdot \sigma^e \cdot n_e \cdot N_{DL} \cdot [1 - f(E_{DL}, T)]$$

- With σ^e = **scattering cross section** (also called **capture cross section**) of the deep level for electrons.

For the other transition rate R^e_u we have to think a little harder. For the electron *trapped* at the deep level to go up to the conduction band it needs a free place up there, hence:

$$R^e_u = r' \cdot (N_{eff}^e - n_e) \cdot n^-_{DL}$$

- With r' = some proportionality constant, *principally different* from r, and $N_{eff}^e - n_e$ = density of free places (which, please remember, aren't *holes*!) in the conduction band.
- Since n_e is much smaller than N_{eff}^e, we may approximate this equation by

$$R^e_u \approx r' \cdot N_{eff}^e \cdot N_{DL} \cdot f(E_{DL}, T)$$

We have not invoked some cross section and thermal velocity here, because the electron now is localized and doesn't move around. We also used a different proportionality constant r' because the situation is not fully symmetric to the reverse process. It is common to call the quantity $e^e = r' \cdot N_{eff}^e$ the **emission probability** for electrons from the deep level.

- The *emission probability* contains the information about the generation of carriers from the deep level; in this it is comparable to the *generation rate* from the valence band for the simple recombination theory considered above.

Now , if we assume that the transitions of conduction band electrons to the deep level and their re-emission to the conduction band are in *local* equilibrium (which does not necessarily entail *total* equilibrium), we have $R^e_u = R^e_d$.

- From this we get – *after a minimal shuffling of the terms* – for the emission probability e^e in *local* equilibrium:

$$e^e = \frac{v \cdot \sigma^e \cdot n_e \cdot [1 - f(E_{DL}, T)]}{f(E_{DL}, T)}$$

Again, as in the case of the generation rate G for direct semiconductors, we may assume *that the emission probability* e^e *is pretty much constant* and this is a crucial point for what follows.

Since we want to find quantities like life times as a function of the density and energy level of the deep level, it is useful to use the mid-band energy level as a reference, and to rewrite the equation for e^e in terms of this mid-band level E_{MB} via the relations

$$\frac{1 - f(E_{DL}, T)}{f(E_{DL}, T)} = \exp - \frac{E_F - E_{DL}}{kT}$$

$$E_{MB} = \frac{E_C - E_V}{2}$$

$$n_i = N_{eff}^{e} \cdot \exp - \frac{E_C - E_{MB}}{kT}$$

- These equations may need a little thought. The first one came up before in a similar way, the second simply defines mid band-gap, and the last one uses the fact that the Fermi energy for intrinsic semiconductors is in mid band gap (at least in a good approximation).

Using these equations, we first rewrite the formula for the density of electrons in the conduction band and obtain

$$n_e = N_{eff}^{e} \cdot \exp - \frac{E_C - E_F}{kT} = N_{eff}^{e} \cdot \exp - \frac{E_C - E_{MB}}{kT} \cdot \exp - \frac{E_{MB} - E_F}{kT} = n_i \cdot \exp \frac{E_F - E_{MB}}{kT}$$

- Putting everything together, we get for the emission probability

$$e^{e} = v \cdot \sigma^{e} \cdot n_i \cdot \exp \frac{E_{DL} - E_{MB}}{kT}$$

- This is the best we can do to describe the traffic of electrons between the deep level and the conduction band.

Next, we do the matching calculation for the *transitions rates with the valence band*, R^{h}_{u} and R^{h}_{d}.

- Except, *we* won't do it. Too boring – everything is quite similar. As the final result for the ***emission probability for the holes***, e^{h}, we obtain exactly what we should expect anyway:

$$e^{h} = v \cdot \sigma^{h} \cdot n_i \cdot \exp \frac{E_{MB} - E_{DL}}{kT}$$

The Net Interband Recombination

We captured the electron and hole traffic betwen a deep level and the conduction or valence band, respectively, with these equations – always for *local* equilibrium of the deep level with the respective band. Now we consider the *interband* generation and recombination rates, G and R.

- This is exactly the same thing as the money traffic form one major bank to another one via an intermediate bank. Each bank can deposit and withdraw money from all three accounts, while the total amount of all the money must be kept constant. If it would be *your* money, you sure like hell would want to and be able to keep track of it. So let's do it with electrons and holes, too.

With G we still denote the rate of electron–hole *pair generation* taking place directly between the bands; by thermal or other energies, e.g. by illumination. It is thus the rate with which electrons and holes are put directly into the conduction or valence band, *no matter what goes on between the deep level and the bands*.

- We may, for some added clarity, decompose G into $G_{perfect}$, the generation always going on even in a hypothetical perfect semiconductor, and G_{ne} for whatever is added in non-equilibrium (e.g. the generation by light). We have $G = G_{perfect} + G_{ne}$.
- After all, before we put in "*our*" deep levels or switched on the light, the hypothetically perfect crystal already must have had some generation and recombination, too (for which $R_{perfect} = G_{perfect}$ must hold). However, we can expect that $R_{perfect}$ is rather small in a perfect indirect semiconductor, which makes $G_{perfect}$ rather small, too.

The rate of change of the electron and hole density in their bands is then the *sum total* of all processes withdrawing and depositing electrons or holes, i.e.

$$\frac{dn_e}{dt} = G_{perfect} + G_{ne} - R_{perfect} + R^e{}_u - R^e{}_d = G_{ne} - (R^e{}_d - R^e{}_u)$$

$$\frac{dn_h}{dt} = G_{perfect} + G_{ne} - R_{perfect} + R^h{}_d - R^h{}_u = G_{ne} - (R^h{}_u - R^h{}_d)$$

- Note that $G_{perfect} - R_{perfect} = 0$ by definition.

Local equilibrium between the bands and the deep level, still not necessarily implying *total* equilibrium, now demands that both dn_e/dt and dn_h/dt must be zero.

- That means that the density of electrons in the conduction band and the density of the holes in the valence band do not change with time anymore. However, that does not mean that they have their *global* equilibrium value, only that we have a so-called **steady state** (in global non-equilibrium) which, on the time scales considered, appears to keep things at a constant value.
- As an example, a piece of semiconductor under constant illumination conditions will achieve a *steady state* in global non-equilibrium conditions. The carrier densities in the bands will be constant, but not at their equilibrium values if light generates electron–hole pairs all the time.
- This gives us the simple equation

$$R^e{}_d - R^e{}_u = G_{ne} = R^h{}_u - R^h{}_d$$

Essentially, this says that the total electron or hole traffic or current [= difference of the partial rates (times elementary charge)] from the conduction or valence band, respectively, to the deep level are identical and equal to the extra band-to-band generation current produced in non-equilibrium for the given material and situation.

- But steady state also implies that there must be an additional recombination exactly equal to G_{ne} and that is of course exactly what the terms $R^e{}_d - R^e{}_u$ or $R^h{}_u - R^h{}_d$ denote: They are identical to the additional recombination rates needed for balancing the additional generation G_{ne}, or simply
- We thus have

$$R^e{}_d - R^e{}_u = R - R_{perfect} = R - G_{perfect} =: U_{DL}$$

- The quantity U_{DL} is exactly analogous to the *difference* **($R - G$)** defined for direct semiconductors.

U_{DL} is also the difference between the recombination to a deep level and the emission from it. For the example considered so far (additional generation via illumination) it must be positive, there is more recombination than generation

- However, our treatment is completely general; U_{DL} can have any value – if it is negative, we would have more generation via deep levels than recombination.
- Of course, U_{DL} makes only sense for global *non-equilibrium* conditions, because for global equilibrium U_{DL} must be zero!

All we have to do now is to express the R^e's with the formulas from above. Inserting the equations for the various R's, the emission probabilities, and setting $\sigma^e = \sigma^h = \sigma$ for the sake of simplicity, we get, after some shuffling of the terms, the *final equation*

$$U_{DL} = \frac{v \cdot \sigma \cdot N_{DL} \cdot (n_e \cdot n_h - n_i^2)}{n_e + n_h + 2n_i \cdot \cosh \dfrac{E_{DL} - E_{MB}}{kT}}$$

- The *cosh* (= hyperbolic cosine) comes from the sum of the two exponential functions. Its value is **1** for $E_{DL} = E_{MB}$; it increases symmetrically for deviations of E_{DL} from the mid-level energy E_{MB}.
- *A chain hanging down from two posts has exactly a **cosh(x)** shape* – that's the way to memorize the general shape of a **cosh** curve. If you want to look more closely at the **cosh** function, activate the link.

- The equation for U_{DL} is quite similar to the one we had for direct semiconductors, as far as the numerator is concerned. We will explore a little more what it implies.
  - For *global* equilibrium, the mass action law $n^e \cdot n^h = n_i^2$ applies, and $U_{DL} = 0$. In other words, there is no *net* recombination, i.e. recombination *in excess* of what is always going on.
  - Without deep levels $U_{DL} = 0$! The recombination rate then is fixed and simply $R_{perfect}$.
  - The recombination rate – everything else being constant – is directly proportional to the *density of the deep levels* and their scattering cross section (or *capture cross section* as it is called in this case).
  - Since the recombination rate is *highest for deep levels exactly in mid-band* (look at the **cosh** function), defects with levels near mid-band are more efficient in recombining carriers than those with levels farther off the mid-band position.

Approximations and the Lifetime τ

- As before, let's look at some special case. Again, we write the carrier densities as $n_{e,h} = n_{e,h}(equ) + \Delta n$ assuming equal Δ 's for electron and holes.
  - This gives us

$$U = v \cdot \sigma \cdot N_{DL} \cdot \frac{[n_e(equ) + \Delta n] \cdot [n_h(equ) + \Delta n] - n_i^2}{n_e(equ) + n_h(equ) + 2\Delta n + 2n_i \cdot \cosh[(E_{DL} - E_{MB})/(kT)]}$$

  - Looking at a **p**-doped semiconductor and only considering the large densities n_h as in the example before, we obtain

$$U = \frac{v \cdot \sigma \cdot N_{DL} \cdot n_h(equ) \cdot \Delta n_e}{n_h(equ) + 2n_i \cdot \cosh[(E_{DL} - E_{MB})/(kT)]}$$

  - Since n_i is also much smaller than n_h**(equ)** , we may neglect the whole **cosh** term, too – as long as $\cosh[(E_{DL} - E_{MB})/(kT)]$ is not large, i.e. for deep levels around mid-band.
  - As a consequence, n_h**(equ)** cancels and we are left with

$$U = v \cdot \sigma \cdot N_{DL} \cdot \Delta n_e$$

- Again, as before, the change in excess minority carrier density is given by $d(\Delta n_e)/dt = -U$, giving

$$\frac{d(\Delta n_e)}{dt} = -v \cdot \sigma \cdot N_{DL} \cdot \Delta n_e$$

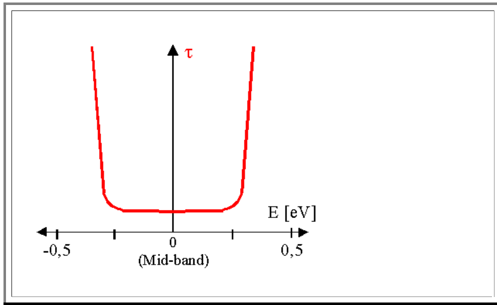
  - The solution of the differential equation now becomes trivial and we have

$$\Delta n_e(t) = \Delta n_e(t=0) \cdot \exp - \frac{t}{\tau}$$

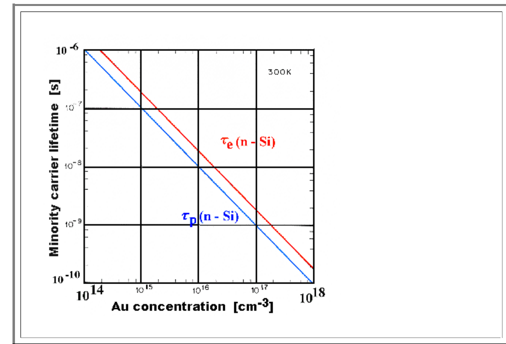
  - with **τ = minority life** time or better **recombination life time** in *indirect* semiconductors defined by

$$\tau = \frac{1}{v \cdot \sigma \cdot N_{DL}}$$

- This is the same equation as before *except that the density of the majority carriers (holes in the valence band for the example) now is replaced by the density of (mid-band) deep levels*.
  - That this formula is a useful approximation is shown in the two illustrations below:



Dependence of the life time on the deep level position relative to the mid level – it is fairly constant (and small) as long as the deep level is approximately in mid band.



Dependence of life time on deep level density – it is linear as predicted. (The red curve is for p-Si, of course.)

- The picture on the right illustrates a sad fact hidden in all these equations: it doesn't take *much* dirt (or **contamination**, to use the proper word) to considerably degrade the life time. Interstitial gold atoms obviously are felt at **10^{14} cm^{-3}** , i.e. at concentrations well below **ppb**.
  - More to Shockley–Read–Hall recombination can be found in an advanced module.