Accumulation

Advanced

- This is the case where an electrical field of arbitrary origin *attracts the majority carriers* .
  - Starting with the Poisson equation for doped semiconductors and all dopants ionized, we have seen that we can approximate the situation by

$$\frac{d^2\Delta E_C}{dx^2} = - e^2 \cdot N_D \epsilon\epsilon_0 \cdot \left(1 - \exp - \frac{\Delta E_C}{kT}\right)$$

  - In contrast to the case of quasi-neutrality, we now have (and the sign is important)

$$- \Delta E_C > kT$$

  - This allows the approximation

$$1 - \exp - \frac{\Delta E_C}{kT} \approx - \exp - \frac{\Delta E_C}{kT}$$

  - and the Poisson equation reduces to

$$\frac{d^2\Delta E_C}{dx^2} = - \frac{e^2 \cdot N_D}{\epsilon\epsilon_0} \cdot \exp - \frac{\Delta E_C}{kT}$$

- Using the Debye length $L_D = \{(\epsilon\epsilon_0\, kT)/(e^2 N_D)\}^{1/2}$, or $N_D = \epsilon\epsilon_0 kT/e^2 L^2{}_D$, the *Poisson equation for accumulation* can be rewritten as

$$\frac{d^2\Delta E_C}{dx^2} = - \frac{kT}{L^2{}_D} \cdot \exp - \frac{\Delta E_C}{kT}$$

  - While this looks like a simple differential equation, it is not all that easy to solve it.

- What we would need first, are defined *boundary conditions* so we can tackle the differential equation. There are no obvious candidates, so we have to think a little harder now.
  - Accumulation means that we have some *surface charge* ρ that we put on the surface of the semiconductor (with our fictive thin insulating layer in between).
  - We thus need to refomulate the differential equation so that *surface charge* can be included. the (not overly obvious) way to do this is to introduce the *electrical field strength* ***E*(*x*)** as a new variable besides ΔE_C.
  - For that we use the relation

$$\frac{d^2\Delta E_C}{dx^2} = \frac{d}{dx}\left(\frac{d\Delta E_C}{dx}\right) = \frac{d}{dx}[e \cdot E(x)] = e \cdot \frac{dE(x)}{d\Delta E_C} \cdot \frac{d\,\Delta E_C}{dx} = e^2 \cdot E(x) \cdot \frac{dE(x)}{d\Delta E_C}$$

  - We also made use of the equaltity $d\Delta E_C/dx = e \cdot E(x)$ with $E(x)$ = field strength.

- Inserting and separating the variables (and omitting the "**(*x*)**" for clarity) gives

$$e^2 \cdot E \cdot dE = \frac{kT}{L^2{}_D} \cdot \exp - \frac{\Delta E_C}{kT} \cdot d\Delta E_C$$

- Tricky, but worth it. Now we can integrate both sides. The integrations run from far inside the bulk , i.e. from $E = 0$, to some value of E, and that means from $d\Delta E_C = 0$ to some corresponding value $d\Delta E_C$.

Omitting the integration (which is trivial), we obtain

$$\frac{E^2}{2} = \left(\frac{kT}{e \cdot L_D}\right)^2 \cdot \left(\exp - \frac{\Delta E_C}{kT} - 1\right)$$

- i.e. an equation relating the amount of band bending at some position x to the electrical field strenght at this point, which is

$$E(x) = \pm \frac{kT}{e \cdot L_D}\left(2\exp - \frac{\Delta E_C(x)}{kT} - 1\right)^{1/2}$$

- For **n**-type semiconductors, which we are considering, ΔE_C is negative and large (i.e. $\Delta E_C >> kT$) - and we may neglect the **– 1**, obtaining

$$E(x) \approx \pm \frac{kT}{e \cdot L_D} \cdot \left(2\exp - \frac{\Delta E_C(x)}{kT}\right)^{1/2}$$

While this is fine, we still don't have the solution we want. We must now remember that there is a *simple relation tying surface charge to volume charge*.

- This is **Gauss law**, stating that the flux of the electrical field through a surface **S** is the integral over the components of E perpendicular to the surface.
- The charge is usually expressed in terms of charge density $\rho(x,y,z)$. Gauss law then states:

$$\iint_S \underline{E} \cdot \underline{n} \cdot da = \frac{1}{\epsilon\,\epsilon} \cdot \iiint_V \rho(x,y,z) \cdot dV$$

- With $\underline{n}$ = normal vector of the surface **S**, **da** = surface increment, **dV** = volume increment. For more details use the link.

For our case it means that we could replace the *total charge* ρ contained in a slice between $x = \infty$ (where there is no charge and the field strength is $E = E_{bulk} = 0$) and x, by a *surface* (or better areal) *charge* $\sigma_{area}(x)$ at x given by

$$\sigma_{area}(x) = \epsilon\epsilon_0 \cdot (E(x) - E_{bulk}) = \epsilon\epsilon_0 \cdot E(x)$$

$$\sigma_{area}(x) = \pm \frac{\epsilon\epsilon \cdot kT}{e \cdot L_D} \cdot \left(2\exp - \frac{\Delta E_C(x)}{kT}\right)^{1/2}$$

The *total* amount of band-bending induced by a real *external surface charge* σ_{ex} is simply $\Delta E_C(x = 0)$ which we call ΔE_C^0 :

$$\Delta E_C^0 = \pm\, 2kT \cdot \ln \frac{\sigma_{ex} \cdot e \cdot L_D}{2^{1/2} \cdot \epsilon\epsilon_0 \cdot kT}$$

- So we have all we need. The **+/-** sign came from the two solutions of the square root; we have to pick the correct one depending on the situation (holes or electrons considered).