Solving the Poisson Equation for pn-Junctions

Advanced

- We have the general equation for the *space charge* $\rho(x)$, and the *Poisson equation*:

$$\rho(x) = e \cdot \{n^h(x) - n^e(x) + N^+{}_D(x) - N^-{}_A(x)\}$$

$$\epsilon\,\epsilon_0 \cdot \frac{d^2V(x)}{dx^2} = -\rho(x)$$

  - $V(x)$ is the built-in potential resulting from the flow of majority carriers to the other side.

- We consider a solution for the following conventions and approximations:
  - The *zero point* of the electrostatic potential is identical to the valence band edge in the **p**-side of the junction shown in the illustration.
  - All dopants are ionized, i.e. $N_A = N^-{}_A = n^h$, and $N_D = N^+{}_D = n^e$. This is always valid as long as the Fermi level is not very close to a band edge.

- For the carrier density we have the general expression

$$n^{e,h} = N_{eff}{}^{e,h} \cdot \exp - \frac{\Delta E}{kT}$$

  - and ΔE was $E_D - E_F$ for electrons and $E - E_A$ for holes.
  - If $E_{D,\,A}$ is a function of x because the bands are bent (while E_F stays constant), we may write the energy difference as $\Delta E = \Delta E^0 + e \cdot V(x)$ with $\Delta E = \Delta E^0$ referring to the situation without band bending.
  - The carrier concentration than becomes

$$n = N_{eff} \cdot \exp - \frac{\Delta E + eV(x)}{kT} = N_{eff} \cdot \exp - \frac{\Delta E}{kT} \cdot \exp - \frac{eV(x)}{kT}$$

$$= N_{A,D} \cdot \exp - \frac{e \cdot V(x)}{kT}$$

  - because the first term gives the concentration for $V(x) = 0$ and that is the dopant concentration in our approximation.
  - We thus have for the carrier concentrations in equilibrium anywhere in the junction:

$$n^h(x) = N_A \cdot \exp - \frac{e \cdot V(x)}{kT}$$

$$n^e(x) = N_D \cdot \exp - \frac{e \cdot \{V(x) - V(n\}}{kT}$$

- As soon as $V(x)$ deviates noticeably from its constant value of **0** or $V(n)$ - in other words: inside the space charge region - the carrier concentrations decrease exponentially from their values N_A or N_D far outside of the **SCR.** We therefore approximate their concentration by

$$n^h = N_A \quad \text{for} \quad x < -d_A$$

$$n^h = 0 \quad \text{for} \quad x > -d_A$$

$$n^e = N_D \quad \text{for} \quad x > d$$

$$n^e = 0 \quad \text{for} \quad x < d$$

- With d_A, d_D = boundaries of the space charge region with $x = 0$ at the geometrical junction

- The space charge then is *only* given by the concentration of the dopants. That's where we could have started right away, just plugging in the usual assumptions. We have

$$\rho = N_A \quad \text{for} \quad -d_A < x < 0$$

$$\rho = N_D \quad \text{for} \quad 0 < x < d_N$$

$$\rho = 0 \quad \text{for} \quad \text{everywhere else}$$

- The Poisson equation then becomes

$$\frac{d^2V}{dx^2} = 0 \quad \text{for} \quad -\infty < x < -d_A$$

$$\frac{d^2V}{dx^2} = +\frac{e}{\epsilon\epsilon_0} N_A \quad \text{for} \quad -d_A < x < 0$$

$$\frac{d^2V}{dx^2} = -\frac{e}{\epsilon\,\epsilon_0} N_D \quad \text{for} \quad 0 < x < d_D$$

$$\frac{d^2V}{dx^2} = 0 \quad \text{for} \quad d_D < x < \infty$$

- In addition we have the boundary conditions:

$$\left. \begin{aligned} V &= 0 \\ \frac{dV}{dx} &= 0 \end{aligned} \right\} \quad \text{for} \quad x = -d_A$$

$$\left. \begin{aligned} V &= V(N) \\ \frac{dV}{dx} &= 0 \end{aligned} \right\} \quad \text{for} \quad x = d_D$$

$$d_A \cdot N_A = d_D \cdot N_D \qquad \text{Charge neutrality}$$

- The solutions are easily obtained, they are

$$V_A(x) = \frac{e}{2\epsilon\epsilon_0} \cdot N_A \cdot (d_A + x)^2 \quad \text{for} \quad -d_A < x < 0$$

$$V_D(x) = V(n) - \frac{e}{2\epsilon\epsilon_0} \cdot N_A \cdot (d_D - x)^2 \quad \text{for} \quad 0 < x < d_D$$

$$V(n) = \frac{e}{2\epsilon\epsilon_0} (N_A \cdot d_A^2 + N_D \cdot d_D^2)$$

- The last equation comes from the condition of continuity at $x = 0$, i.e. $V_D(x = 0) = V_A(x = 0)$.

The two limits of the space charge region, d_A and d_D , as well as the field strength $E = -dV/dx$ in the **SCR** thus could be calculated if we would know $V(n)$.

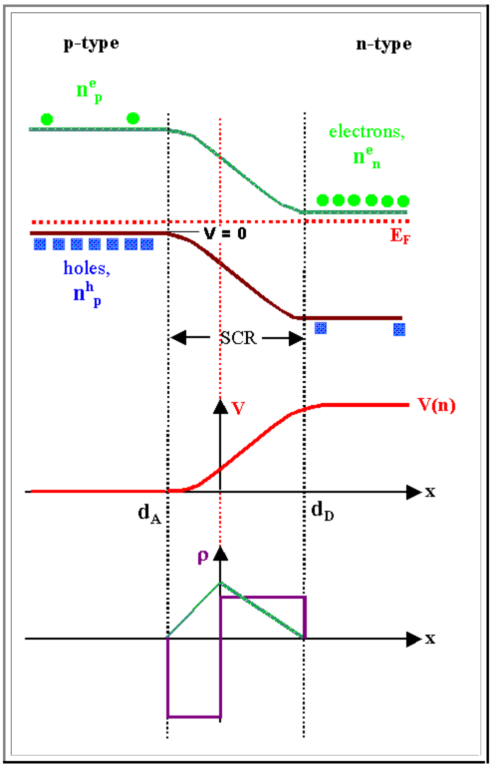
- ***V*(n)**, of course, is the difference of the potential across the **SCR** and thus identical to **1/e** times the difference of the Fermi energies before contact in thermal equilibrium, we have

$$V(n) = -\frac{E^n_F - E^p_F}{e}$$

- If we superimpose an external voltage U, $V(n)$ becomes (watch out for the correct sign!) .

$$V(n) = \frac{E^n_F - E^p_F}{e} \pm eU$$

The following illustration shows the whole situation in one drawing.



We now can express the width d_{SCR} of the space charge region as

$$d_{SCR} = d_A + d_B = \frac{1}{e}\left(2\,\epsilon\,\epsilon_0 \cdot [\Delta E_F + e \cdot U_{ex}] \cdot \left(\frac{1}{N_A} + \frac{1}{N_D}\right)\right)^{1/2}$$

- ΔE_F refers to the the difference of the Fermi energies before the contact and U_{ex} is the external voltage.