2.4 Operators representing physical properties

The Energy-Operator: Hamilton Operator, Hamiltonian

$$A|\psi\rangle = \frac{\hbar}{i}\frac{\partial}{\partial t}|\psi\rangle = E\psi\rangle \tag{2.7}$$

$$\text{Eigenvector:} \qquad e^{i\omega t} \tag{2.8}$$

$$\text{Eigenvalue:} \qquad E = \hbar\omega \tag{2.9}$$

A synonym for the Hamilton Operator is time evolution operator. The meaning of this expression will be explained later.

The Momentum-Operator:

$$A|\psi\rangle = P_x|\psi\rangle = \frac{\hbar}{i}\frac{\partial}{\partial x}|\psi\rangle = p_x|\psi\rangle \tag{2.10}$$

$$\text{Eigenvector:} \qquad e^{ik_x x} \tag{2.11}$$

$$\text{Eigenvalue:} \qquad p_x = \hbar k_x \tag{2.12}$$

The Eigenstates are plane waves. In this representation the Eigenvectors just show the space dependence. Since according to Eq. (2.9) each state has a time dependence, we find that each Momentum-Eigenvector reflects the properties of a plane wave.
The wave length is

$$\lambda = \frac{2\pi}{|k_x|} \tag{2.13}$$

The Space-Operator:

$$A|\psi\rangle = X|\psi\rangle = x|\psi\rangle = a|\psi\rangle \tag{2.14}$$

i.e we find

$$(x-a)|\psi\rangle = 0 \tag{2.15}$$

For $x \neq a$ follows $|\psi\rangle = 0$. In order to get a state, which can be normalized, we must set:

$$|\psi\rangle = \delta(x-a) \tag{2.16}$$

Thus we find for the expectation value of the space operator:

$$\langle x\rangle = \langle\psi|X|\psi\rangle = \int_{-\infty}^{+\infty}\psi^*(x)x\psi(x)dx = \int_{-\infty}^{+\infty}x||\psi||^2dx = \int_{-\infty}^{+\infty}xP(x)dx \tag{2.17}$$

$$P(x)dx = ||\psi||^2dx \tag{2.18}$$

is obviously the probability for finding a particle in the interval $(x, x+dx)$.