2.3 Calculation of expectation values

Let be:

Operator	Eigenvalue	Eigenvector
A	a_k	f_k (orthonormal)

The representation of a general quantum mechanical state in the space of the above defined Eigenvectors is:

$$|\psi\rangle = \sum c_k |f_k\rangle \tag{2.1}$$

For the length we find:

$$\langle\psi|\psi\rangle = \sum c_l \langle f_l|c_k|f_k\rangle = \sum c_k^* c_k \tag{2.2}$$

Since $c_k^* c_k$ is the fraction of the state k to the whole system, for a system with one particle we must find:

$$\langle\psi|\psi\rangle = 1 \tag{2.3}$$

According to axiom 4 we get:

$$\langle a\rangle = \sum a_k c_k^* c_k \tag{2.4}$$

Easily we find:

$$\langle\psi|A|\psi\rangle = \sum a_k c_l^* \langle f_l|c_k|f_k\rangle = \sum a_k c_k^* c_k = \langle a\rangle \tag{2.5}$$

This is the abstract representation of an expectation value, since it is independent of the choice of the orthonormal basis. If ψ is not orthonormal, Eq. (2.5) has to be replaced by

$$\frac{\langle\psi|A|\psi\rangle}{\langle\psi|\psi\rangle} = \langle a\rangle \tag{2.6}$$