1.5 Component representation of linear operators

Each operator can be defined in an abstract way by,

$$y = Ax \qquad , \tag{1.32}$$

e.g.

- A: Rotation of an angle of 45^o around the x-axis
- A: Time developing operator of a quantum mechanical wave function

For using this operator in calculations, we have to define, how this operator acts on each vector x. Taking the component representation of x and y

$$x = \sum_i x_i |e_i\rangle \qquad \text{and} \qquad y = \sum_i y_i |e_i\rangle \tag{1.33}$$

we find

$$y = \sum_i y_i |e_i\rangle = Ax = \sum_i x_i A|e_i\rangle \qquad . \tag{1.34}$$

Multiplication with the Bra-vector leads to

$$y_j = \sum_i \langle e_j | y_i | e_i \rangle \sum_i \langle e_j | A | e_i \rangle x_i \qquad . \tag{1.35}$$

- The components $m_{i,j} = \langle e_j | A | e_i \rangle$ thus define the operator A.
- The matrix M consisting of the components $m_{j,i}$ is thus equivalent to the operator A.
- We call M a representation of the operator A.
- Every linear function can be represented by a matrix.

Example:

Choosing the vector space of square integrable functions, the set of functions e^{ikx} serves as an orthonormal basis. Every vector (function) f may be represented as components of the functions e^{ikx}. This is the Fourier-Analysis (Fourier-Transformation) of the function f. The result of an operator A on a square integrable function is thus defined, when the components $m_{i,j} = \langle e_j | A | e_i \rangle$ are known.

All results, proofed here for matrixes, are valid for linear operators as well.

REMARKS:

- The functions e^{ikx} are not exactly square integrable.
- The "indices" i, j for plane waves e^{ikx} are continuous. The sum over i has therefore to be replaced by an integration over k.