5.3 The Fermi statistics

The Fermi statistics is a direct consequence of Eq. (5.3) assuming the independence of the occupation of individual states. Using the definition

$$A_j = \prod_{i \neq j} W_{k,i} \qquad , \tag{5.4}$$

the probability for occupying the state j with an electron is proportional to

$$A_j \exp\left(-\frac{E_j - \mu}{kT}\right) \qquad . \tag{5.5}$$

The electron in state j adds the energy $(E_j - \mu)$ to the complete energy.
The probability that the state j is not occupied is proportional to

$$A_j \qquad , \tag{5.6}$$

since in this case the energy zero is added to the complete energy.
This are all possibilities for a Fermion to occupy a state; consequently we find the probability for occupying the state j:

$$W_j = \frac{A_j \exp\left(-\frac{E_j - \mu}{kT}\right)}{A_j + A_j \exp\left(-\frac{E_j - \mu}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_j - \mu}{kT}\right)} \qquad . \tag{5.7}$$

The Fermi statistics describes the probability to occupy a one electron state in an ensemble. Essential for this calculation are independent electrons since for the above calculation we need that A_j is independent of the occupation of state j.