3.7 The Pauli principle

The choice of the representation of the system must have no influence on physical parameters, i.e.

$$||\psi||^2 = \langle n_1, n_2, n_3, ...|n_1, n_2, n_3, ...\rangle = \langle n_2, n_1, n_3, ...|n_2, n_1, n_3, ...\rangle \tag{3.23}$$

the exchange of two indices is not important for expectation values. We find for the wave function

$$|\psi\rangle = |n_1, n_2, n_3, ...\rangle = \pm|n_2, n_1, n_3, ...\rangle \tag{3.24}$$

Only symmetric or antisymmetric wave functions are allowed when exchanging two indices.
Within quantum field theory one can show that particles with an integer number as spin have symmetric state functions while particles with a half number of the spin have always antisymmetric state functions.
Thus electrons have an antisymmetric state function, which is extremely important for a many particle state

$$|\psi\rangle = |n_1, n_2, n_3, ...\rangle = -|n_2, n_1, n_3, ...\rangle = 0 \tag{3.25}$$

Choosing a state in which the electron 1 and the electron 2 occupy the same single particle state, i.e. $n_1 = n_2$, we find

$$|\psi\rangle = |n_1, n_1, n_3, ...\rangle = -|n_1, n_1, n_3, ...\rangle = 0 \tag{3.26}$$

This state is not existing!
The postulation of an antisymmetric wave function for the many particle state of a system is nothing else but the Pauli principle. The Pauli exclusion law is therefore a direct consequence of the fact that quantum mechanical particles are indistinguishable.
This exclusion law is fundamental for many properties of solids. Expressions like

- an occupied state
- a fully occupied band
- an empty band
- a hole
- the Fermi energy
- ...

are directly coupled to the Pauli principle.

(On the other hand, the world would be quite dark if, e.g., photons would obey the Pauli principle. In addition it would be very complicated for light to reach the earth from the sun and the amount of X-rays would be extremely high.)

Summing up, the Axioms of quantum mechanics do not *explain what* a quantum mechanical system is, they only allow to describe a quantum mechanical system correctly. The isometry between the mathematical description of the model and the experimental results by measuring justifies the axioms. But there is no principle difference to simpler models and their mathematical formulation since we expect in both cases that our imagination and the corresponding models reflect at least a part of the reality. This will be clarified by the following example:

"The blue balls are Sodium (Na), the red ones are Chloride (Cl); the springs are the bindings; thus we can describe phonons by the following model.....

In quantum mechanics this illustrative feature is missing. Modeling starts directly with the mathematical formulation but in the end this approach is much more powerful than all illustrative models.