3.2 Conserved Properties/Constants of Motion

Only those measurements will characterize the state of a system, which will lead to the same result in successive measurements.
For this set of measurements we will show that the corresponding operator A commutes with the Hamiltonian H of the system:

$$[A, H] = 0 \quad . \tag{3.7}$$

In this case the Eigenvector system of A is identical to that of the Hamiltonian, i.e. the measurement prepares a state

$$\psi(t, \vec{r}) = f_m e^{\frac{iE_m t}{\hbar}} \quad ; \tag{3.8}$$

only the phase changes as a function of time. A successive measurement will find always the same Eigenvalue. The energy and the expectation value of the operator A are thus always measurable at the same time.

The state of as system is defined completely if all expectation values of those operators are known which commutate with the Hamiltonian.

More (meaningful, useful) information can not be gathered about a quantum mechanical system.
This is the complete description of a quantum mechanical system.

Example 1: The potential well / Free Particle

The Hamiltonian is

$$H = \frac{p^2}{2m} + const. \quad . \tag{3.9}$$

We find

$$[p, H] = 0 \quad . \tag{3.10}$$

The momentum operator is the only operator which commutates with the Hamiltonian; consequently the energy state is defined completely by the Eigenstate of the momentum operator.

REMARK: There is no Hamiltonian which commutates with the space operator (each particle has kinetic energy). Therefore the knowledge about the location of a particle never is a meaningful description of a quantum mechanical system.

Example 2: The Hydrogen atom

The Hamiltonian is

$$H = \frac{p^2}{2m} - \frac{const.}{r} \quad . \tag{3.11}$$

The following operators commutate with this Hamiltonian

1. the operator L^2, with $\vec{L}$ being the rotation momentum operator,
2. the operator L_z and
3. the operator S_z, with $\vec{S}$ being the spin operator.

The state of an Hydrogen atom is thus completely defined by the quantum numbers

H	L^2	L_z	S_z
n	l	m	s

These quantum numbers are an adequate description of an electronic state of a Hydrogen atom (But who can for example imagine the Eigenvector of the rotational momentum operator?). These information allow to calculate the atomic orbitals. BUT: the electron is not somewhere in this orbital with a well defined probability. It is everywhere at the same time. Only the measurement of the position of the electron forces the electron to occur at one special place with a defined probability.