2.8 The Commutator

We define the commutator of two operators A and B by

$$[A, B] = AB - BA \tag{2.23}$$

This operator defines the difference between the results of two following measurements when the order of both measurements is switched; e.g. for real numbers the commutator is always zero. For quantum mechanic nonzero commutators are essential. Thus we had to introduce operators, since the commutator of operators is usually not zero.

Examples:

1. The rotation around two different axes. This rotation is described by matrices (= Operators)
2. $[P, X] = (PXf - XPf) = \frac{\hbar}{i} f$, leading to $[P, X] = \frac{\hbar}{i}$

Defining the uncertainty ΔA of a physical quantity by

$$\Delta A = \sqrt{\left\langle (A - \langle A \rangle)^2 \right\rangle} = \sqrt{\langle A^2 \rangle - \langle A \rangle^2} \tag{2.24}$$

one can proof the following relation:

$$\Delta A \Delta B \geq ||\langle [A, B] \rangle|| \tag{2.25}$$

If $[A, B] = 0$ holds, there exists a commonly used system of Eigenvectors. Eq. (2.25) is a formulation of this fact (Eq. (2.20)) which does not depend on the representation of the operators.