2.7 The Particle-Wave-Dualism

The most important example of two physical properties which can not been measured at the same time are location and momentum of a particle. The unitary transformation which couples both operators is the Fourier-Transformation. We will not use Eq. (2.19) to get an exact result but will just calculate an approximation.
We look at a state that is a superposition of a couple of free electrons with similar momenta:

$$\psi(x) = \int_{p_x-\frac{\Delta p}{2}}^{p_x+\frac{\Delta p}{2}} e^{\frac{ixp_x}{\hbar}} dx = \frac{2\hbar}{x} \sin\frac{x\Delta p}{2\hbar} e^{\frac{ixp_x}{\hbar}} \tag{2.21}$$

The uncertainty of the momentum is obviously Δp. $||\psi||^2$ has its first zeros at $\frac{x\Delta p}{2\hbar} e^{\frac{ixp_x}{\hbar}}$. It's main weight has the width π. This leads to the uncertainty relation

$$\Delta x \Delta p = h \tag{2.22}$$

The more exactly the position of a particle is defined the less information one has concerning its momentum (and vice versa).