2.6 The Uncertainty relation

We will investigate what happens when performing two measurement one after another. Both measurement shall belong to different operators:

Operator	F	G
Eigenvalue	f_n	g_m

The first Measurement: Expectation value of F

The system makes a decision for the state f_i

Questions:

- how long will the state exist?
- what will happen when repeating the measurement of F?

If the state will exist at least for a short time, the following measurement of F will lead to the same state f_i.

$\Rightarrow$ We prepared a state.

The following Measurement: Expectation value of G

Applying the unitary transformation U, we get:

$$f_i = \sum_m U_{im} g_m \tag{2.19}$$

The system decides in favor of the state g_m with a probability $||U_{i,m}||^2$.
Two different cases exist, leading to extremely different results for the measurement:

1. Both Operators have the same system of Eigenvectors:

In this case we find

$$||U_{im}||^2 = \delta_{im} \qquad ; \tag{2.20}$$

the particle will not change its state. A third measurement, now again for Eigenvalue of the operator F, would give the same result as the first measurement. Both measurable quantities can thus be "sharp" at the same time.

Example:

The free electron:

1. We measure the momentum P
2. We measure the energy $\frac{P^2}{2m}$

Both operators are linear and Hermitian. Measuring the momentum p_i in the following measurement we will find the Eigenvalue $\frac{p_i^2}{2m}$ for the energy.

2. Both Operators have different systems of Eigenvectors:

In this case, the system has to decide in favor of a new state which is an Eigenvector of the operator G. The formerly measured Eigenvalue of the operator F is probably changed.