Available online at www.sciencedirect.com

ScienceDirect

Energy Procedia 92 (2016) 255 – 264

Energy Procedia

6th International Conference on Silicon Photovoltaics, SiliconPV 2016

# Linear-response description of the series resistance of large-area silicon solar cells: Resolving the difference between dark and illuminated behavior

Jan-Martin Wagner[a,*], Andreas Schütt[b], Jürgen Carstensen[a], Rainer Adelung[a]

[a]*Cristian-Albrechts-Universität zu Kiel, Technische Fakultät, Kaiserstraße 2, 24143 Kiel, Germany*
[b]*CELLOscan GbR, Gravelottestr. 5, 24116 Kiel, Germany*

**Abstract**

Directly from luminescence images it can be shown that, for constant average injection (lumped dark current) and for not-too-large lateral voltage differences, besides the sign, the current flow direction doesn't play any role for the voltages present, so the series resistances in the dark and under illumination are the same. This fits to the results of a linear-response based series resistance description, treating lateral voltage differences on large-area silicon solar cells in linear order in the series resistance as deviation from the case of zero resistance. In this approach it is found that for constant lumped dark current, emitter and grid of a large-area solar cell can be described as a passive network. Therefore, no difference occurs in the voltage distribution caused by inward and outward currents except for the sign. This contradicts several literature works reporting a smaller lumped series resistance of silicon solar cells in the dark than under illumination. However, we show that this contradiction is just a result of the series resistance definition applied in the respective works or that it can be the result of unsuitable measurement conditions. In a numerical modeling of a large-area silicon solar cell as a 1D distributed structure, using exactly the same parameters as Araújo *et al*. [IEEE-TED 33 (3), 391–401 (1986)] but calculating the lumped series resistance from the integrated Joule losses, we obtain completely different results than Araújo *et al.*: Under short-circuit condition, the series resistance stays constant, and there is no difference between the open-circuit and dark series resistance; the latter show the same dependence on the diode current density.


Peer review by the scientific conference committee of SiliconPV 2016 under responsibility of PSE AG.

*Keywords:* series resistance; injection dependence; lateral voltage offsets; lumped Joule losses; solar cell modeling; linear-response theory; luminescence-based series resistance imaging; dark series resistance; illuminated series resistance

* Corresponding author. Tel.: +49-431-880-6181; fax: +49-431-880-6124.
*E-mail address:* jwa@tf.uni-kiel.de


Peer review by the scientific conference committee of SiliconPV 2016 under responsibility of PSE AG.
doi:10.1016/j.egypro.2016.07.072

## 1. Introduction

The equivalent lumped series resistance $R_s$ of a solar cell has at least one, if not two physical meanings: First, it allows to express the sum of all ohmic heating losses (Joule losses) in the solar cell as

$$P_s = R_s I_{ext}^2 , \quad (1)$$

and second, under certain conditions (cf. [1]) it provides the voltage difference between the external contacts and the effective internal voltage related to replacing the whole solar cell by an effective ideal diode (or two diodes in case of the two-diode model) according to the equivalent circuit. The latter role of the series resistance – providing the local voltage – is of importance for all local solar cell efficiency analyses, carried out *e.g*. in any optimization procedure related to changes in solar cell manufacturing.

Various series resistance measurement prescriptions, related to different $R_s$ concepts, can be found in the literature. One of the latter has been introduced to treat the well-known variation of the lumped series resistance with the operating conditions (cf., *e.g*., [2] and references therein) directly in the framework of the equivalent circuit. In this modeling, the external-current-density dependent series resistance $r_s(J_{ext})$ is defined by the following expression for the effective diode voltage of the equivalent circuit [$J_{ext}$ is the external current density, here taken as positive for inward (dark) current and negative for outward (photo-)current, and $U_{ext}$ is the terminal voltage]:

$$U_D = U_{ext} - r_s(J_{ext})\, J_{ext} . \quad (2)$$

In the literature, several works report a smaller lumped series resistance of silicon solar cells in the dark than under illumination. For example, based on the above-given expression for the diode voltage, Eq. (2), and a numerical modeling of a large-area silicon solar cell as a 1D distributed structure, Araújo *et al*. [2] have explicitly displayed $r_s(J_{ext})$ for the dark, open-circuit, and short-circuit cases in dependence on the external current density; always their dark value is the lowest of all. However, we have found that there is no such difference between the series resistance behavior in the dark and for the open-circuit case. In this contribution we want to resolve this contradiction, and we provide a consistent description via an appropriate replacement for the standard equivalent circuit model.

**Nomenclature**

LR-$R_s$ Linear-response series resistance

## 2. Experimental and theoretical basics: the LR-$R_s$ method

Parts of the basics of the LR-$R_s$ method have already been published [3–5]; here we present it in a consistent manner. After considering the lumped series resistance $R_{s,cell}$ for a fixed dark current, we explain how to obtain the distributed series resistance $R_s^{distr}(x,y)$ from an auxiliary function $R(x,y)$ and discuss its dark current dependence.

### *2.1. The lumped series resistance*

The most reliable measurement of the lumped series resistance of a solar cell is obtained by the comparison of two illuminated *I–U* curves for different illumination strengths [6]; the relative error of this method can be considerably decreased by using multiple *I–U* curves, i.e. several illumination levels [7]. In this method, those points of the *I–U* curves are compared where the solar cell is under identical injection conditions as measured by the dark current flowing. From the differences in the relevant external cell currents, $\Delta I_{cell}$, and the corresponding voltages, $\Delta U_{ext}$, the series resistance is determined as

$$R_{s,cell} = \Delta U_{ext} / \Delta I_{cell} . \quad (3)$$

In this paper, based on photoluminescence measurements using red light for illumination, we replace the condition of "equal dark current" by the equivalent one of "equal average luminescence signal", as illustrated in Fig. 1.

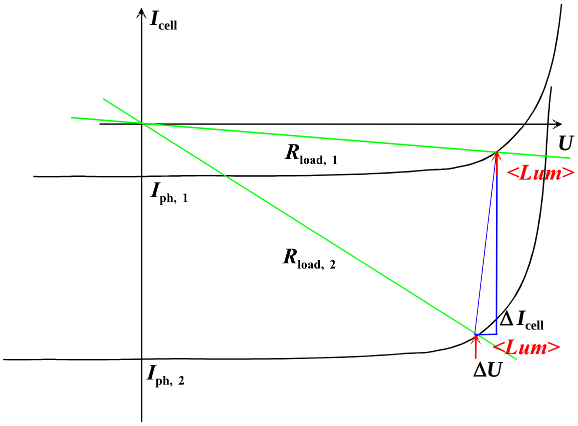


Fig. 1. Illustration of series resistance determination using identical average luminescence values (symbolized by the red arrows labeled "<Lum>") to determine identical injection conditions, as given by identical differences of the external current to the full photocurrent.

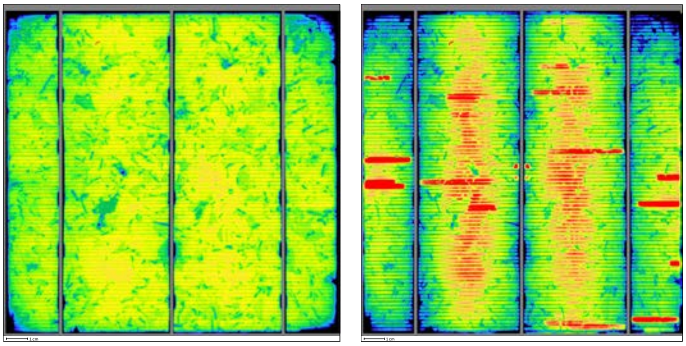

Fig. 2. Example for lumped series resistance determination using identical average luminescence values to find identical injection conditions: Both images have an average of 2992 counts. Left: open circuit ($U$ = 595,5 mV); right: current extraction ($U$ = 580,5 mV, $I_{cell}$ = –6.5 A).

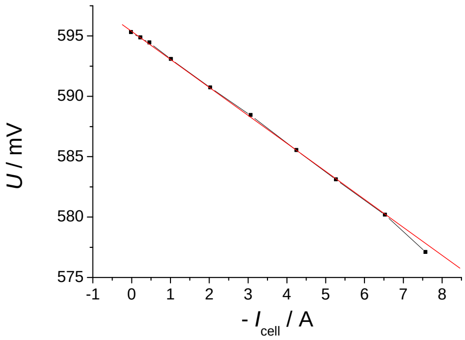


Fig. 3. Variation of the cell voltage with extracted current for identical dark current, as determined from average luminescence values measured for the cell of Fig. 2. The slope of the observed linear I–U relation is the lumped series resistance, Rs,cell, for the given dark current, cf. Eq. (3).

For various loads, ranging from extraction of a large current of several amps to open-circuit condition, the illumination is adjusted to obtain the same average counts in the luminescence images. For each image taken, the external cell current and the external cell voltage are measured. Two such images are shown in Fig. 2, and the obtained perfect linear relation between cell current and voltage for a set of 10 measurements (using 10 different illumination levels and extracted currents up to 7.5 A) is shown in Fig. 3; only at the highest current, a slight nonlinearity occurs. This experimentally found linear relation between voltage and current justifies our approach using the average luminescence signal as reference for the series resistance measurement.

### *2.2. The distributed local series resistance*

From the measured luminescence images, a map of the distributed series resistance can be obtained. The method used here was described earlier [3, 4], it uses a linear response description of the solar cell and does not rely on the model of independent diodes. "Linear response" here means that a change in the local voltage somewhere on the solar cell, $\Delta U_{loc}(x,y)$, occurring due to a variation of the total external current, $\Delta I_{ext}$, is expressed as being proportional to this variation, the proportionality factor being an auxiliary function $R(x,y)$:

$$\Delta U_{loc}(x,y) = R(x,y)\, \Delta I_{ext} \,. \tag{4}$$

The proportionality factor $R(x,y)$ is defined by this proportionality; its meaning is the total resistance of the solar cell, containing both series and diode resistance. The change in the local voltage can easily be obtained from the ratio of two luminescence images $\Phi_1(x,y)$, $\Phi_2(x,y)$, corresponding to the two different external currents (cf. [8]):

$$\Delta U_{loc}(x,y) = U_{therm} \ln\left[\frac{\Phi_1(x,y)}{\Phi_2(x,y)}\right] , \tag{5}$$

where the thermal voltage $U_{therm}$ may include a diode (non-)ideality factor, if necessary. For laterally homogeneous diode resistance (which is found in most "sufficiently regular" solar cells, cf. [9–11]) this lateral voltage variation is determined by the lateral series resistance. Directly at the external contact, located at position $(x_0,y_0)$, the voltage variation is largest, and the lateral series resistance is negligible. Going into the area of the solar cell, the voltage change becomes smaller, since the lateral series resistance increases. Therefore, comparing the local voltage change to the one at the external contact provides information about the lateral series resistance. In linear-response description one has for the lateral series resistance (here called $R_s^{distr}$ for reasons becoming clear only later) that

$$R_s^{distr}(x,y) = \frac{\Delta U_{loc}(x_0,y_0) - \Delta U_{loc}(x,y)}{\Delta I_{ext}} = R(x_0,y_0) - R(x,y) \cdot \tag{6}$$

Of course, since a solar cell under operation is a forward-biased p–n junction, linear response behavior is not a trivial thing to expect but has to be experimentally verified. For practical measurements, one of the luminescence images frequently is taken under open circuit condition, having a voltage distribution of $U_0(x,y)$. For this case, Eq. (4) can be written as

$$U(x,y,I_{ext}) - U_0(x,y) = R(x,y)\, I_{ext} \quad \text{or} \tag{7a}$$
$$U(x,y,I_{ext}) = U_0(x,y) + R(x,y)\, I_{ext} \,. \tag{7b}$$

To separate the lateral series resistance from the diode resistance, for the measurement under external current flow the illumination strength is varied from the open-circuit one so that the lumped dark diode current is the same for both measurements; this basically eliminates the influence of the non-linear diode behavior. For example, in the case of current extraction the illumination strength is increased as much as to provide the additional photocurrent (as discussed in Sect. 2.1 for the accurate determination of $R_{s,cell}$; cf. Fig. 1). For any given external current, according to Eqs. (6) and (7b) the local voltage distribution can be written as

$$U(x,y,I_{ext}) = U(x_0,y_0,I_{ext}) - R_s^{distr}(x,y)\, I_{ext} + U_0(x,y) - U_0(x_0,y_0)\ , \quad (8)$$

i.e., the lateral voltage drop (from a given point to the contact) is not only determined by the voltage drop at the lateral series resistance, but also by the voltage distribution under open-circuit condition (which for multicrystalline cells comes mainly from inhomogeneities of the dark current [as can be expressed by a $J_0$ distribution], acting on the series resistance network of the cell [5, 12]). In other words: The part $R_s^{distr}(x,y)\, I_{ext}$ is only the contribution of the external current to the voltage distribution, it adds to the one for the open-circuit case corresponding to the relevant injection (as given by the lumped dark diode current).

In addition to the distributed series resistance there is always a homogeneous contribution that shows no spatial dependence, here it is called $R_s^{nondistr}$. It is given by the difference between the lumped resistance (determined as described in Sect. 2.1) and the arithmetic mean of the image of the distributed series resistance. Two such images of the distributed series resistance (for the cell of Fig. 2), differing in the extracted current, are shown in Fig. 4. These linear response series resistance images (which we from now on call “LR-$R_s$” images) have the same arithmetic mean value, and their difference is just noise (not shown). Therefore, for this cell and at the given injection, the distributed series resistance does not depend on the strength of the extracted current.

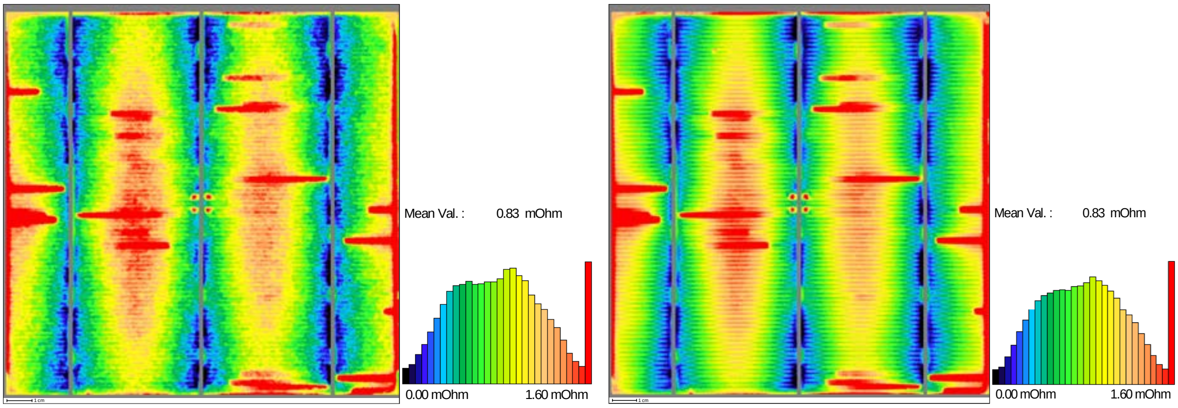


Fig. 4. Distributed series resistances images of the cell shown in Fig. 2, calculated according to Eq. (4). Left: cell current –1.03 A; right: cell current –6.5 A.

### 2.3. The injection (dark diode current) dependence of the series resistance

The series resistance of a solar cell is not constant but depends on the injection condition [3–7, 9–11]. The global and local series resistance measurement is therefore repeated for various injections, measured by the current injected across the diode, $I_D$, and (for reasons becoming clear below) converted to the lumped diode resistance, $R_D$, by

$$\frac{1}{R_D} := \frac{dI_D}{dU_D} \approx \frac{q}{kT} I_D . \quad (9)$$

The variation of the lumped cell series resistance with the injection is shown in Fig. 5. The same curve is shown in the left part of Fig. 6 as a black line, together with the average distributed series resistance value (arithmetic mean), obtained from the relevant images (as those shown in Fig. 4). The difference between these two values is shown in the right part of Fig. 6, it is essentially constant (mind the scaling of the ordinate). Therefore, one can conclude that the lumped series resistance of the cell can be written as

$$R_{s,cell} = R_s^{nondistr} + \left\langle R_s^{distr} \right\rangle , \quad (10)$$

the angle brackets (here and elsewhere in this paper) indicating the arithmetic mean value of the respective image.

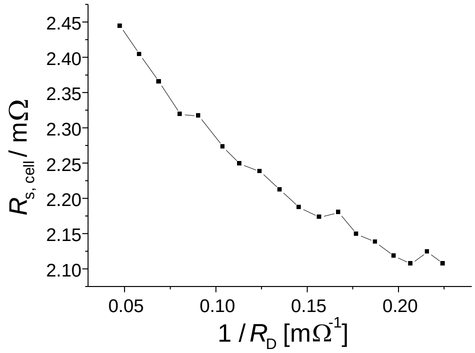


Fig. 5. Lumped series resistance versus the inverse of the lumped diode resistance (i.e., lumped dark current) for the cell shown in Fig. 2.

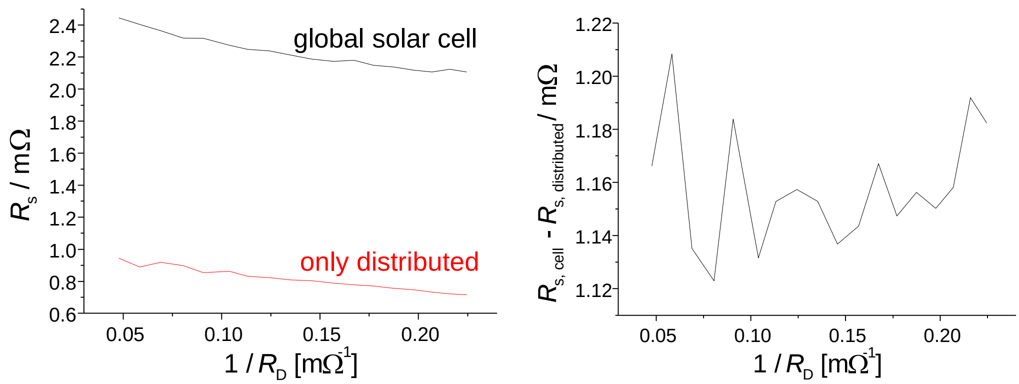


Fig. 6. Left: variation of the lumped and the average distributed series resistance value with the inverse of the lumped diode resistance [cf. Eq. (9)] for the cell shown in Fig. 2. Right: the difference of global and average distributed series resistance value is essentially constant.

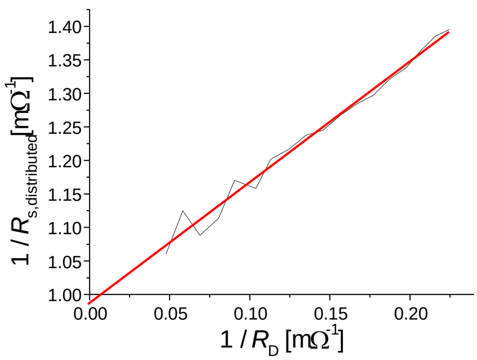


Fig. 7. Injection dependence of the inverse of the average distributed series resistance value (arithmetic mean) for the cell shown in Fig. 2. As a direct experimental result, a linear relationship is found, which can be described by Eq. (11).

Plotting the inverse of the distributed series resistance versus the injection (as given by $R_D^{-1}$), a linear relation is obtained as a direct experimental result, shown in Fig. 7. The intersection of this straight line with the ordinate gives the inverse of the average distributed series resistance for infinite diode resistance and is therefore called $R_{s,\infty}^{-1}$. The slope of the straight line being referred to as $g$, one can therefore express the observed linear relationship as

$$\frac{1}{\left\langle R_s^{\mathrm{distr}} \right\rangle} = \frac{1}{R_{s,\infty}} + g\frac{1}{R_D}. \tag{11}$$

Therewith, Eq. (10) becomes

$$R_{s,\mathrm{cell}} = R_s^{\mathrm{nondistr}} + \frac{1}{1/R_{s,\infty} + g/R_D}. \tag{12}$$

The value of $g$ depends on the geometry of the distributed series resistance network, therefore this letter was chosen.

This result is theoretically well understood (cf. [9–11]) and found since many years for nearly all monocrystalline solar cells and good multicrystalline solar cells. The expression for the distributed series resistance, Eq. (11), has a rather simple interpretation: Under forward bias, lateral current flow is reduced by bypassing current through the distributed diodes to the back side, reducing ohmic losses in the emitter and the grid. We note that this strong, but predictable dependence of the series resistance on the injection condition has important consequences for the description of the *I–U* curve of a solar cell, cf. [1] for details.

### *2.4. Equivalent circuit and the dark vs. illumination comparison*

One of the main results of the linear response series resistance described above is that for constant lumped dark current, emitter and grid can be described as a passive network, the resistance of which, however, is not constant but varies with the effective diode resistance. According to Eq. (12), this can be expressed graphically by the following modified equivalent circuit (Fig. 8) which includes the variation of the lumped series resistance resulting from the switch-on of the diodes. As a consequence of emitter and grid behaving as a passive network, no difference occurs in the voltage distribution caused by inward and outward currents except for the sign. This contradicts several works in the literature (cf. [2] and references therein) reporting a smaller lumped series resistance of silicon solar cells in the dark than under illumination. In the remainder of this work, we address this (as it will turn out: seeming) discrepancy.

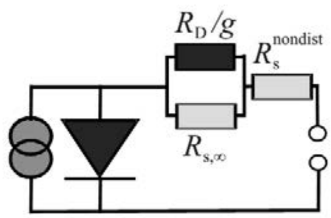


Fig. 8. Modified equivalent circuit, consistent with the variation of the series resistance due to the varying diode resistance (i.e., dark-current dependence of the series resistance).

## 3. Experimental results

We have performed electro- (EL) and photoluminescence (PL) measurements of an mc-Si solar cell for constant average injection (lumped dark current) and for not-too-large lateral voltage differences (injected and extracted current 3 A), cf. Figs. 9(a), (c).

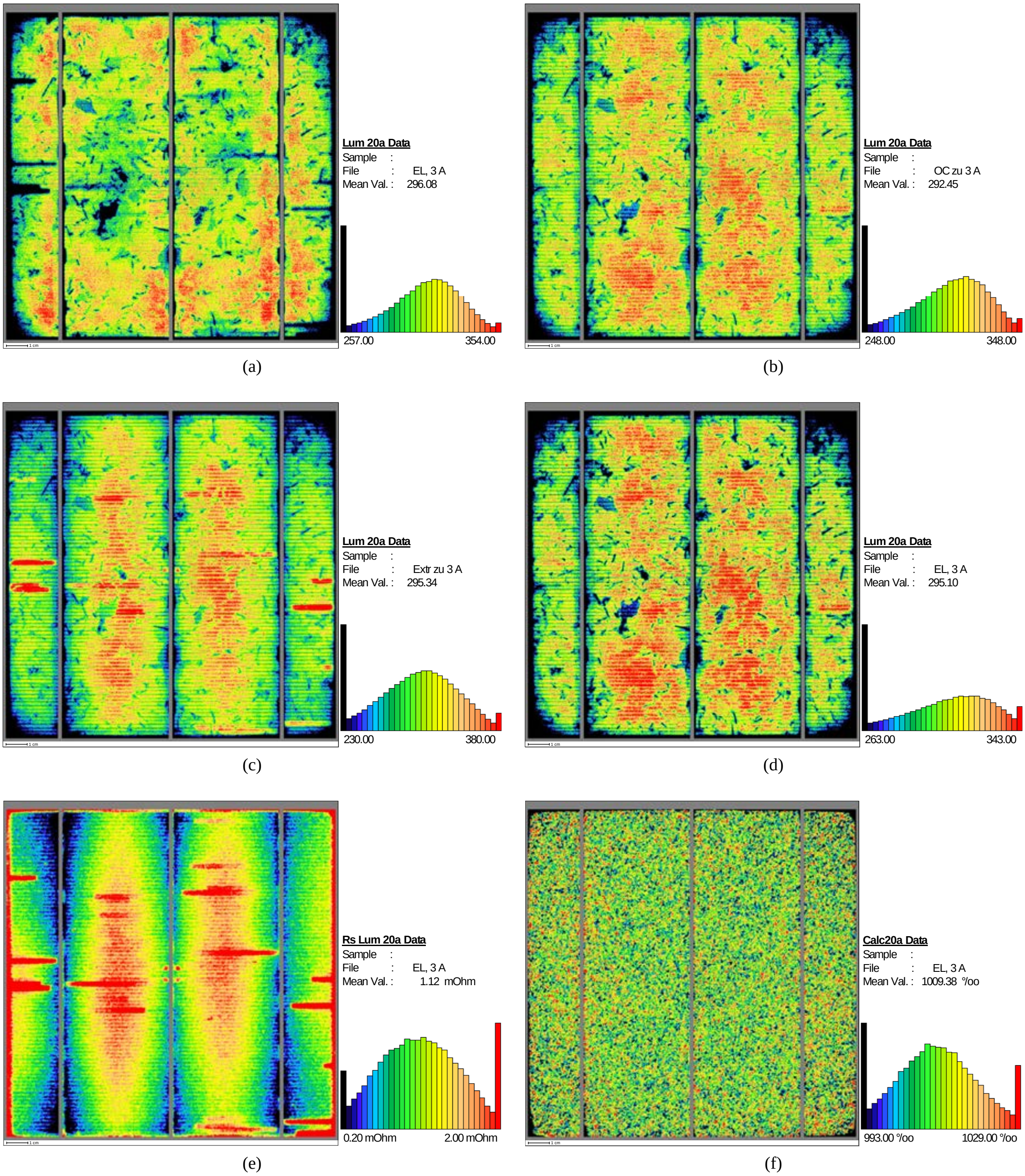


Fig. 9. Luminescence images of the cell shown in Fig. 2 at a fixed injection. (a) EL, $I_{ext}$ = 3 A; (b) under open circuit and 0.5 sun illumination; (c) PL under one sun illumination and with $I_{ext}$ = –3 A; (d) geometric mean of (a) and (c). Additionally shown: (e) LR-$R_s$ series resistance image obtained from (a) and (b); (f) ratio of (d) and (b).

Combining these images by taking their geometric mean (i.e., taking the square root of the product of the pixel intensities), an image results, Fig. 9(d), which is identical to the directly measured open-circuit image taken for the same average injection (in praxis, for the same arithmetic mean value of the luminescence intensity), Fig. 9(b): The ratio of the latter two, Fig. 9(f), is just noise at an average value of 1.01 (the deviation from 1.0 results from the small difference in the mean values). This means that the series resistance effects present in the EL and PL image have cancelled out – which is only possible if they are just of opposite sign but otherwise identical. In fact, calculating a linear-response based series resistance image either from using the PL image under current extraction and the oc image or from combining the EL the and oc image yields the same result; the one obtained from EL is shown in Fig. 9(e).

## 4. Numerical results

We solved numerically the same 1D problem as Araújo *et al*. [2], using their parameter values, but calculated the lumped series resistance from the integrated Joule losses, cf. Eq. (1), i.e. we used a completely model-free approach to numerically obtain the series resistance. Our results, presented in Fig. 10, correspond directly to the three curves shown by Araújo *et al*. (in their Fig. 4). Our results are in perfect agreement with Eq. (12), applying Eq. (9) for the translation of diode currents to diode resistances.

There are two findings which deviate completely from the results of Araújo *et al.*: First, our short-circuit value of the series resistance doesn't increase; it stays perfectly constant. Second, in the onset range of their decrease there is no difference between our open-circuit and dark $r_s$ curves; only in the region of steepest decay a slight difference occurs. (We note that here, as in the work of Araújo *et al*., "open circuit" does not refer just to the open-circuit point of the *I*–*U* curve, but to the open-circuit region of the characteristic; cf. the derivation of Eq. (20) in [2].)

We attribute these deviations from the results of Araújo *et al*. to their series resistance modelling, which is based on $r_s(J_{ext})$ as mentioned above, cf. Eq. (2). Their model is just an *I*–*U* curve description without a sound physical justification, in contrast to the determination of the series resistance from the integrated Joule losses which is capable to give correct results for all operating conditions.

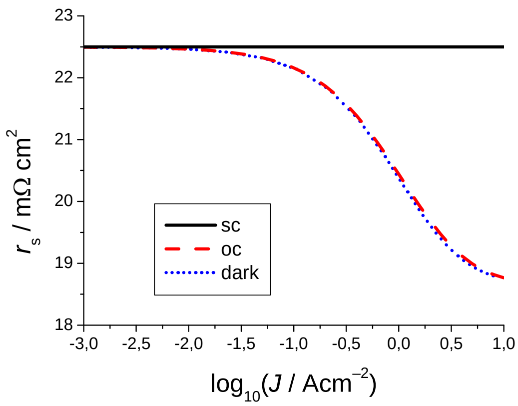


Fig. 10. Results of a numerical calculation of the lumped series resistance for a 1D distributed model, using the same parameters as in [2] but calculating the lumped series resistance from the integrated Joule losses. For the short-circuit curve (sc), the abscissa is the photo-current density $J_{ph}$, while for the open-circuit (oc) and dark curve, the abscissa is the dark diode current density $J_D$.

## 5. Summary and conclusions

At constant injection (dark diode current), emitter and grid of a large-area silicon solar cell can be described as a passive network. As a consequence, there is no difference in the series resistance behavior of large-area silicon solar cells in the dark and under illumination; previously found differences stem from an inadequate definition of the series resistance or unsuitable measurements. (There is a relevant exception: Shading due to the front metallization indeed leads to a difference in the lumped series resistance [13], however this does not contradict the fact that emitter and grid of a large-area silicon solar cell can be described as a passive network.)

The variation (decrease) of the series resistance along any current–voltage characteristic results primarily from the switching-on of the diodes; the lumped series resistance $R_{s,cell}$ is a function of the lumped dark current $I_D$ only, i.e. $R_{s,cell} = R_{s,cell}(I_D)$. This occurs already in linear order in the series resistance and is not related to current crowding; this regime can still be described by a modified equivalent circuit.

As a consequence of the dark-current dependence of the series resistance, only series resistance measurement methods that work at a constant dark current can yield reliable results; this holds, *e.g.*, for the double (or multiple) illumination method.

From a theoretical point of view, only the definition of the lumped series resistance on the basis of the integrated Joule losses yields a value which is physically meaningful for all points along the *I–U* curve and for arbitrary photo-current densities.

## Acknowledgements

J.-M. Wagner acknowledges financial support from the Chair of Functional Nanomaterials, Technical Faculty, University of Kiel. The authors are indebted to O. Breitenstein, Max-Planck-Institute of Microstructure Physics, Halle (Saale), Germany, for a critical reading of the manuscript as well as valuable discussions.

## References

[1] Carstensen J, Wagner J-M, Schütt A, Föll H. Relation between local and global *I–U* characteristics: Restrictions for and by series resistance averaging In: Proc. 29th European Photovoltaic Solar Energy Conference, Amsterdam, The Netherlands (2014); contribution 2BV.8.21.
[2] Araújo GL, Cuevas A, Ruiz JM. The effect of distributed series resistance on the dark and illuminated current–voltage characteristics of solar cells. IEEE Transactions on Electron Devices 1986;33(3):391–401.
[3] Wagner J-M, Carstensen J, Berhane A, Schütt A, Föll H. Series resistance analysis with the shaded luminescence technique. In: Proc. 26th European Photovoltaic Solar Energy Conference, Hamburg, Germany (2011); contribution 2BV.2.51.
[4] Wagner J-M, Hoppe M, Schütt A, Carstensen A, Föll H. Injection-level dependent series resistance: Comparison of CELLO and photoluminescence-based measurements. Energy Procedia 2013;38:199–208.
[5] Carstensen J, Wagner J-M, Schütt A, Föll H. Ohmic loss analysis for lateral balancing currents by CELLO and photoluminescence measurements. In: Proc. 29th European Photovoltaic Solar Energy Conference, Amsterdam, The Netherlands (2014); contribution 2BO.2.6.
[6] Wolf M, Rauschenbach H. Series resistance effects on solar cell measurements. Advanced Energy Conversion 1963;3:455–79.
[7] Fong KC, McIntosh KR, Blakers AW. Accurate series resistance measurement of solar cells. Progress in Photovoltaics: Research and Applications 2013;21(4):490–9.
[8] Kasemann M, Reindl LM, Michl B, Warta W, Schütt A, Carstensen J, Contactless Qualitative Series Resistance Imaging on Solar Cells. IEEE Journal of Photovoltaics 2012;2(2):181–3.
[9] Carstensen J, Schütt A, Föll H. CELLO local solar cell resistance maps: Modeling of data and correlation to solar cell efficiency. In: Proc. 22nd European Photovoltaic Solar Energy Conference, Milan, Italy (2007); contribution 1CV.1.34.
[10] Carstensen J, Schütt A, Föll H. Modelling of the distributed serial grid resistance: Verification by CELLO measurements and generalization to other resistance mapping tools. In: Proc. 23rd European Photovoltaic Solar Energy Conference, Valencia, Spain (2008); contribution 1CV.1.38.
[11] Carstensen J, Abdollahinia A, Schütt A, Föll H. Characterization of the grid design by fitting of the distributed serial grid resistance to CELLO resistance maps and global IV curves. In: Proc. 24th European Photovoltaic Solar Energy Conference, Hamburg, Germany (2009); contribution 1CV.4.33.
[12] Carstensen J, Wagner J-M, Schütt A, Adelung R. Solving the code of series resistance on large area solar cells: Average and local power losses of external and lateral balancing currents. In: Proc. 31st European Photovoltaic Solar Energy Conference, Hamburg, Germany (2015); contribution 2BV.8.22.
[13] Cuevas A, Araújo GL, Ruiz JM. Variation of the internal series resistance with the operating conditions of a solar cell: dark and illuminated cases. In: Proc. 5th European Photovoltaic Solar Energy Conference, Athens, Greece (1983); pp. 114–8.