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Efficient focusing with a concave lens based on a photonic crystal with an unusual effective index of refraction

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This work reports measurements with a concave lens based on a photonic crystal (PC) structure, which was designed to have an effective index of refraction $n_{eff} < 1$ or even < 0, and which is intended as a model system for future down-scaled optical elements based on PCs with an unusual effective index. The dimensions of the model PC were optimized for experiments in the microwave regime around 10 GHz. Calculations for a material with no losses allowed to select the wavelengths for which the lens could be expected to behave as a homogeneous meta-material with an unusual effective refractive index. The field distribution behind the lens was measured, and good focusing efficiencies for $n_{eff} < 1$ were found for perfect and strongly disturbed PC's in reasonable accordance with the predictions. The (down-scaled) model system investigated thus can serve as a reference for testing PC-based optical elements made from materials which so far elude reliable predictions, e.g. doped semiconductors, nanorod assemblies, or meta-materials with anisotropic behavior.

Photonic crystals (PC) are meta-materials that offer novel ways for localizing and propagating electromagnetic waves including light [1]. The essential required ingredient for making a PC is a periodicity of the refractive index. This may generate forbidden states for photons in the PC, prohibiting propagation of radiation with a certain wavelength, and it also may generate unusual dispersion functions in the allowed bands. As a consequence, periodic dielectric structures may have an effective index of refraction that is less than unity, or even negative, and this effect may be used to construct novel optical elements with enticing features [2]. However, the present theoretical understanding of these effects is based on considering mostly the real part of the complex dielectric function, and its applicability to materials like porous semiconductors, which are the materials of choice for PCs, is therefore not entirely clear.

The general goal of this work is to demonstrate first both theoretically and experimentally the focusing characteristics of periodic or quasi-periodic dielectric structures with a negligible imaginary part of the dielectric function in the microwave spectral region, and then to use this well-characterized (and scaled-down) structure as a model system for testing materials with more complex dielectric functions (or magnetic susceptibilities), like porous semiconductors or nanorod based meta-materials.

In this paper we present first results on measurements and theoretical calculations of the focusing effect of a concave lens based on a two-dimensional PC. The PC is made from circular alumina rods arranged in a perfect or strongly disturbed cubic lattice. The diameter and length of the rods is $d = 1$ cm and $l = 50$ cm, respectively, and the lattice constant of the PC is $a = 2.8$ cm. For certain wavelengths, comparable to the lattice constant, such a PC will behave, in a good approximation, as an optically homogeneous material for which one index of refraction can (i) be well defined, and (ii) have an unusual value of $n_{eff} < 1$ or even $n_{eff} < 0$. Geometrical optics then dictates that a concave lens built from such a material would have to focus radiation, and we prove that this is indeed the case. As the dimensions of the PC's parameters suggest, the measurements

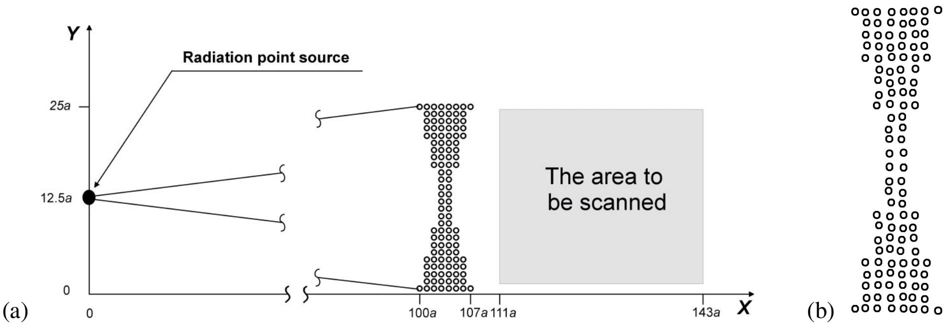

Figure 1 a) Top view of the measured lens that consists of periodically arranged alumina rods. b) A displacement of up to 30% from their regular position should (and does) not destroy the focusing effect of the lens. In this case the structure will be called: "amorphous lens".

were done in the microwave regime. However, due to the scalability of the Maxwell equations, the same phenomena will be observed for wavelengths lying in the optical region if the dimensions are reduced accordingly.

Describing a (highly inhomogeneous) PC by an effective index of refraction has been done before. Lalanne [3] discussed how effective medium theory (EMT) can be used under certain restrictions to "homogenize" a PC, and Notomi [4] claims that a PC can be assigned an effective index of refraction for quasi wave vectors around the Γ point, i.e. the center of the Brillouin Zone. The (local) effective refractive index in this case will be determined by the local radius of the equi-frequency surfaces (EFS). Close to the band edge the EFS is rather spherical, allowing to define just one index of refraction for all directions; the PC then behaves as a continuous isotropic material.

The determination of the form and the radii of EFS is not a simple task. For small PC's like the concave lens in our case, it may even become meaningless, since the reciprocal lattice and the Brillouin zone are no relevant parameters of the system any more. Moreover, for heavily distorted PCs, resembling rather an amorphous structure than a crystal, the band-structure approach is questionable or must at least be reconsidered.

We proposed earlier a simpler approach, based on the use of a virtual probe medium [5], that allows to easily determine if an effective refractive index is actually well-defined, and what value it should be assigned in this case. The frequency ranges where this condition is met are indeed near the van Hoove singularities around the Γ point [6], here a (large) PC will behave as a homogeneous material at least in a decent approximation. The effective index of refraction, calculated using the probe medium approach, will generally be smaller than unity for the following (approximate) values of a/λ: $a/\lambda \in (0.67; 0.92) \cup (1; 1.1)$ for the TM mode and $a/\lambda \in (0.6; 0.68) \cup (0.7; 0.86) \cup (1; 1.15)$ for the TE mode. If this is true, then a structure based on such a PC that resembles a concave lens would have to focus radiation with wavelengths lying in the respective intervals. However, for the reason given above, the total effect of this lens on an impinging wave front must be individually calculated, and the results demonstrate that the lens does not always behave as a homogeneous medium in the predicted ranges.

The shape of a generic concave lens is approximated by the structures presented in Fig. 1; calculations and measurements were made for perfect periodicity and for a strongly disturbed ("amorphous") arrangement of the rods

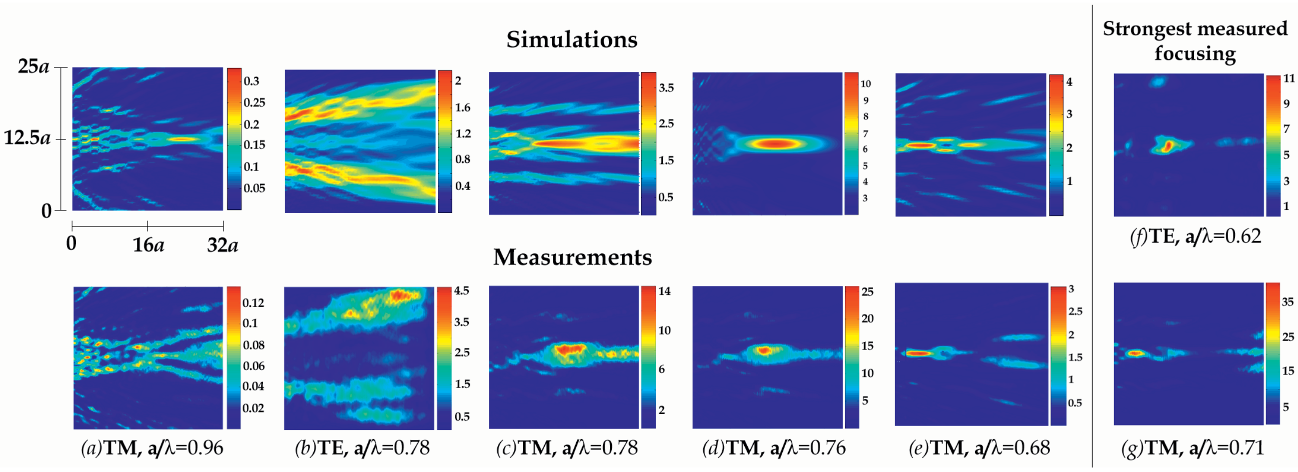

Figure 2 Simulated and measured power gain of the concave lens for the TE and TM polarizations and various configurations. Note the difference in the power gain scale; for details refer to the text.

(Fig. 1b) and for the wavelengths in the 5 cm region. A dipole was placed at a distance of 100*a*, acting essentially as a point source. The electromagnetic field was scanned over an area of $32a \times 25a$ in a plane perpendicular to the dielectric rods in the middle of the lens, i.e. at a height of 25 cm; beginning at 4*a* behind the edge of the lens as shown in Fig. 1a. The electromagnetic field intensity distribution $E^2(x, y)$ was first taken in free space, i.e. without the lens, yielding $E_0^2(x, y)$, and then with the lens, yielding $E^2(x, y)$. The focusing efficiency of the lens is then defined as the ratio $T = E^2(x, y)/E_0^2(x, y)$; it is thus the power gain in the presence of the lens.

Experiments were started with a wavelength not contained in the intervals stated; a well-defined index of refraction should thus not exist. No focusing was observed as expected, cf. Fig. 2a with $a/\lambda = 0.96$ and TM polarization. As soon as the wavelengths are approached where the PC is expected to have a well-defined $n_{\text{eff}} < 1$, the first signs of focusing for both polarizations can be observed, cf. Fig. 2b and Fig. 2c for TE and TM polarizations for the same value of $a/\lambda = 0.78$. While focussing is observed for the TM mode, the lens acts as a “beam-splitter” for the TE mode. Beam splitting is mostly observed for TE polarization and $n_{\text{eff}} < 1$; while focusing is only observed (and predicted) in the small interval $a/\lambda \in (0.6; 0.68)$. Optimizing the wavelength for the TM polarization resulted in clear focusing of the beam as shown in Fig. 2d.

Based on the theoretical predictions, an “amorphous” lens was constructed by displacing the cylinders randomly up to 30% from their original places as shown in Fig. 1b. First results with this “amorphous” lens are shown in Fig. 2e for $a/\lambda = 0.68$. Compared to a perfect lens, the power gain is reduced, but the image point is clearly expressed and definite focussing occurs.

The best focusing conditions observed for perfect lenses are presented in Fig. 2f and Fig. 2g for TE and TM polarizations, respectively. Values for the intensity gain as large as 11 for TE and 35 for TM polarizations could be measured, proving that the concept of optical components made from photonic crystals with an unusual effective index of refraction is sound and may find applications.

Note that the simulations for the individual cases are generally in very good agreement with the experiments, lending credibility to the approach, but that on occasions the lens performs considerably better than calculated.

Some earlier works also discussed possible lenses made from PCs [7–9], but good focusing with unusual n_{eff} has not been shown before. Gupta and Ye [7], e.g., proposed and measured a convex lens based on an alumina rod PC, but with an $n_{\text{eff}} > 1$. For our lens, having a radius of curvature of 62.5 cm, best focussing conditions correspond to an $n_{\text{eff}} = (-0.3 \pm 0.5)$, and its focussing efficiency is at least three times better than the lens discussed in [7].

In conclusion, it has been shown that the lens made from a rather perfect material with respect to the imaginary part of the dielectric function behaves essentially as calculated. A scaled-down version for wavelengths in the mm region (corresponding to about 100 GHz) thus can be used to easily test materials with non-negligible imaginary part ε'' of the dielectric functions like doped semiconductors, which are the prime candidates for achieving the ultimate goal of novel optical elements in the IR or visible range. In particular, semiconductors like Si can be used for the proposed experiments, where different doping levels provide an adjustable range of ε'' values, or even allow to produce periodicities in ε'' different from that of the real part ε'. Moreover, preliminary calculations showed that replacing just one rod of the lens may alter the behavior significantly, thus allowing to test (meta-)materials only available in small quantities like nanorod “powders” with negative magnetic susceptibilities, ferromagnetic fluids with adjustable magnetic field induced anisotropies, or porous semiconductors with a strongly anisotropic ε' tensor and unclear ε'' [10].

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