SOLVING THE CODE OF SERIES RESISTANCE ON LARGE AREA SOLAR CELLS: AVERAGE AND LOCAL POWER LOSSES OF EXTERNAL AND LATERAL BALANCING CURRENTS

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ABSTRACT: Lateral balancing currents are an immanent feature of solar cells with an inhomogeneous distribution of photocurrent $I_{ph}(x,y)$ and/or diode current $j_{01}(x,y)$ leading to additional power losses and thus to a significant increase of difficulty to analyze local efficiencies of solar cells. Due to fundamental physical restrictions like charge conservation and having a potential distribution across a 2D grid network, astonishingly only the histogram information (no local information!) of images of recombination strength and series resistance is needed to calculate all relevant average information. This is completely analogous to the technique of frequency analysis used for code breaking used for more than 1000 years where just a histogram analysis allows to identify the meaning and the position of all (most) characters within a text. In this contribution, the theory and quantitative results for several inhomogeneous solar cells with different kinds of lateral balancing currents are presented.

Keywords: photoluminescence, series resistance, lateral balancing currents, open-circuit voltage, multicrystalline silicon

1 INTRODUCTION

Mapping the local power distribution of solar cells, their local current and voltage contributions, and the underlying defect structures is presently the top topic in solar cell characterization. Since currents are extensive parameters, it is in principle not difficult to add them up, calculate averages and to generate local maps, representing their distribution across a solar cell. The real challenge is the distributed network of local (non-linear) diodes being connected by local series resistances. In several papers [1 - 4] we have shown that the approximation of this distributed 2D network up to linear order in the series resistances is necessary and sufficient for "normal" solar cells (those which you can buy/sell) to allow for a description of the solar cell components using the standard equivalent circuit (cf. left or right side of Fig. 1) and to fulfill several implicit expectations for mapping of the local contribution for each of the average parameters (photo current, diodes, series resistance) showing up in the equivalent circuit:

- The distributed network of local diodes, local photo current sources, and local series resistances behaves with respect to external current and external voltage as if one global resistance is in series to the global diode and the global photo current.
- For each of the global (average) parameters unambiguous averaging is possible, independent of the local distribution of the other parameters.

Extracting averages from local maps which have a reasonable meaning for the global *I*–*V* curve implies two additional properties:

- The effect of a mapped property is proportional to the area on the solar cell which shows a certain value of that property.
- The effect on the global *I*–*V* curve is proportional to the value of the property.

So in principle only linear parameters can be analyzed using the concepts presented in this paper, which is seemingly in contradiction to the non-linear diode characteristics. Actually at each point along the I-Vcurve local and global diode characteristics can be linearized, allowing for the concepts discussed above but implying fully theoretically understood and experimentally proved injection dependent global and local series resistances [2, 4].

Lateral balancing currents are an immanent feature of

solar cells with an inhomogeneous distribution of photocurrent $I_{ph}(x,y)$ and/or diode current $j_{01}(x,y)$, leading to additional power losses and thus to a significant increase of difficulty to analyze local efficiencies of solar cells. Due to fundamental physical restrictions like charge conservation and having a potential distribution across a 2D grid network, astonishingly only the histogram information (no local information!) of images of recombination strength and series resistance is needed to calculate all relevant average information. For quantitative analysis of local series resistances this concept has been published years ago and is routinely used for a quantitative evaluation of CELLO and luminescence measurements [2, 4 (and references therein)]. Here we will present the theory and quantitative results for several inhomogeneous solar cells with different kinds of lateral balancing currents.

The approaches are completely analogous to the technique of frequency analysis used for code breaking used for more than 1000 years where just a histogram analysis allows to identify the meaning and the position of all (most) characters within a text [5]. It is quite counterintuitive that by losing all information about the position of characters within a coded text just by counting the number of occurrence the meaning of individual characters can be identified more easily than by their lateral relation within a text. The fundamental knowledge that in each language certain characters (e.g. 'e', 's', 't') show up more often and that the ratio for the occurrence of such characters is quite fix (for a text long enough), allows often to identify the language a coded text is written in, and to identify enough characters that after substituting back into the coded text the remaining decoding is quite trivial.

For the quantitative analysis the procedure has been published already, only the results will be used in the paper to analyze ohmic losses related to lateral balancing currents. In this paper we will use diode current maps $j_{01}(x,y)$ extracted from open circuit luminescence maps to calculate lateral balancing currents. The first step is to model the (hidden) relation between certain averages of $j_{01}(x,y)$ map data and lateral balancing currents (corresponding to the knowledge of fixed ratios for the occurrence of certain characters within a text). The second part describes how such $j_{01}(x,y)$ map averages efficiently can be extracted just from the histogram data of that map.

2 THEORY

Under open circuit condition in strong enough forward direction the global the I-V characteristic reads

$$I_{\text{ext}} = 0 = J_{01} e^{\frac{qU_{oc}}{kT}} - I_{Ph} = A_{sc} j_{01} e^{\frac{qU_{oc}}{kT}} - A_{sc} i_{Ph} (1)$$

i.e.
$$I_{Ph} = A_{sc} j_{01} e^{\frac{qU_{oc}}{kT}}$$
(2)

Here j_{01} is the average diode current density and i_{Ph} the average photo current density and A_{sc} the area of the solar cell. For the moment we will ignore the lateral variation of $i_{Ph}(x, y)$ since it is much smaller than the lateral variation of $j_{01}(x, y)$ (especially for using red light for measuring the open circuit luminescence as used for the experiments presented in this paper). The area $A_{sc,d}$ with $j_{01}(x, y) < j_{01}$ we call drains, the remaining areas are sources. The average of all $j_{01}(x, y) < j_{01}$ we call $j_{01,d}$ and the average of the remaining area $j_{01,s}$. Let $\alpha_d = A_{sc,d}/A_{sc}$.

Now we can easily calculate the lateral balancing current

$$I_{\text{lat}} = (j_{01,s} - j_{01})(1 - \alpha_d) A_{sc} e^{\frac{qU_{oc}}{kT}}$$
(3a)

resp.
$$-I_{\text{lat}} = (j_{01,d} - j_{01})\alpha_d A_{sc} e^{\frac{dU_{oc}}{kT}}$$
 (3b)

Dividing these equation by the left, resp. right hand side of Eq. (2) we get

$$\frac{I_{\text{lat}}}{I_{Ph}} = \frac{j_{01,s} - j_{01}}{j_{01}} (1 - \alpha_d)$$
(4a)

resp.
$$-\frac{I_{\text{lat}}}{I_{Ph}} = \frac{j_{01,d} - j_{01}}{j_{01}} \alpha_d$$
 (4b)

We now already introduced most parameters of the equivalent circuit in Fig. 1 showing two solar cells in parallel which are a simplified model for describing quantitatively the effect of lateral balancing currents flowing from the good into the bad parts of a solar cell. The remaining parameters are just two series resistances together with geometrical scaling parameters so that their net resistance equals the global series resistance of the solar cell. Ignoring these resistances we have exactly the situation described by Eqs. (1) to (4). Since power losses are related to voltage losses $U_{Rs} = \left(\frac{R_s}{1-\alpha_d} + \frac{R_s}{\alpha_d}\right)I_{lat}$, and the voltage is already a function in linear order in R_s the change in the lateral current due to R_s would already be in second order in R_s and thus will (can) be ignored.

In order to calculate the deviation from U_{ext} to the average potential applied to the diode in the source region we just have to multiply the lateral current with the corresponding resistance

$$\frac{R_s}{1-\alpha_d} \frac{I_{\text{lat}}}{I_{Ph}} = \frac{j_{01,s} - j_{01}}{j_{01}} R_s$$
(5a)

The corresponding deviation from U_{ext} to the average potential applied to the diode in the drain region is

$$-\frac{R_s}{\alpha_d} \frac{I_{\text{lat}}}{I_{Ph}} = \frac{j_{01,d} - j_{01}}{j_{01}} R_s$$
(5b)



Fig. 1: Simplified equivalent circuit for an inhomogeneous solar cell, divided into a "good" part 1 (low recombination; "source") with an area fraction $(1 - \alpha)$ and a "bad" part 2 (high recombination; "drain") with an area fraction α . Due to the difference in dark current, a lateral balancing current I_{lat} flows in addition to the external current I_{ext} .

Please note that all area-related factors (α_d) cancel out in the right hand side of the Eqs. (5a) and (5b). This is a general feature and a must since we calculate (relative) voltages, which are intensive parameters. In strong contrast the left hand sides of the Eqs. (5a) and (5b) depend strongly on the area, e.g. lateral balancing currents flowing out of a large good area do change the potential much less than the same lateral current flowing into a small bad region. So for deriving and proper interpretation of the Eqs. (5a) and (5b) a strict distinction between current and current density is obviously necessary. Lateral ohmic losses are driven by currents, not by current densities; this is true for most standard series resistance losses as well.

The Eqs. (5a) and (5b) hold for averages of parameters extracted from maps. Before discussing the translation of these equations to calculate corresponding local map data we will show how the average data can easily be calculated just from histogram data of $j_{01}(x, y)$ maps. Let H(j) be the histogram data, so obviously

$$j_{01} = \frac{\int_{-\infty}^{+\infty} j H(j) dj}{\int_{-\infty}^{+\infty} H(j) dj}.$$

In the next step we calculate

$$j_{01,s} = \frac{\int_{j_{01}}^{+\infty} j H(j) dj}{\int_{j_{01}}^{+\infty} H(j) dj} \quad \text{and} \quad j_{01,d} = \frac{\int_{-\infty}^{j_{01}} j H(j) dj}{\int_{-\infty}^{j_{01}} H(j) dj}$$

This approach can be applied iteratively to get a finer distribution of lateral balancing currents, where the averages of the iteration before are the limits of the corresponding integrals for the following iteration. As an example we just split up the source region into finer subsource and sub-drain:

$$j_{01,ss} = \frac{\int_{j_{01,s}}^{+\infty} j H(j) dj}{\int_{j_{01,s}}^{+\infty} H(j) dj} \quad \text{and} \quad j_{01,sd} = \frac{\int_{j_{01}}^{j_{01,s}} j H(j) dj}{\int_{j_{01}}^{j_{01,s}} H(j) dj}$$

Defining additionally the area fraction

$$\alpha_{dd} = \frac{\int_{j_{01}}^{j_{01,s}} H(j) dj}{\int_{j_{01}}^{+\infty} H(j) dj}$$

one finds

$$\frac{R_s}{(1-\alpha_d)(1-\alpha_{dd})} \frac{I_{\text{lat}}}{I_{Ph}} = \left(\frac{j_{01,s} - j_{01}}{j_{01}} + \frac{j_{01,ss} - j_{01,s}}{j_{01,s}}\right) R_s$$
(6a)

and
$$\frac{R_s}{(1-\alpha_d)\alpha_{dd}} \frac{J_{\text{lat}}}{J_{Ph}} = \left(\frac{j_{01,s}-j_{01}}{j_{01}} + \frac{j_{01,sd}-j_{01,s}}{j_{01,s}}\right) R_s$$
 (6b)

Completing the set of 4 equations with those belonging to the original drain region we divided the solar cell into four regions for which the integral lateral balancing current and the average related voltage losses are known. Especially the right hand side can easily be translated back into a map information, e.g. by assigning the right hand side value of Eq. (6b) to all data points in the $j_{01}(x, y)$ maps with values between j_{01} and $j_{01,s}$. The resulting map is close to a local information of relative voltage losses related to lateral balancing current flow, but is only correct "on average". Multiplying with the global photo current I_{Ph} (used for measuring the luminescence maps) it represents the absolute voltage losses related to lateral balancing current flow. Additionally dividing by I_{lat} a local resistance map $R_{\text{lat}}(x, y)$ is calculated. Please note that in this analysis no free parameter shows up, which can/must be adapted. For the experimental results presented in this paper the solar cell has been iteratively divided into 16 parts. This was enough to get reasonable local information and is a very robust approach since it reduces the information of more than 1 million data points in the luminescence maps to just 16 average values. The transition to real local information has to overcome three problems:

- 1. Up to now no local information about the local series resistance losses have been taken into account.
- 2. Transferring the average information back into map data, due to the discretization into 16 average values strong local discontinuities show up.
- 3. In several parts of the map regions with large positive lateral balancing currents are found close to regions with large negative balancing currents. Here the lateral balancing currents will just take the short cut not leading to ohmic losses as on average for lateral current flow across the solar cell.

Several approaches to overcome these problems have been tested. A very simple procedure, leading already to very good results will be discussed shortly: Just smooth the resulting map strongly using a median filter approach. Afterwards multiply the resulting smoothed map with the series resistance map $R_s(x, y)$ divided by the global R_s . Since in our approach the average of the $R_s(x, y)$ map always equals the global R_s the average of the original map does not change as long as both maps are uncorrelated. Dividing the solar cell just in 16 parts typically does not lead to significant correlation between the maps.

That this approach solves problems 1 and 2 is obvious. Problem 3 gets improved, since close to the interfaces of regimes with different lateral balancing currents by averaging the differences get smaller; having two regimes, one with positive, the neighboring with negative lateral balancing current at the interface the values level out to zero. This cancelling out does not mean that no balancing currents exist, but that these balancing currents do not contribute to ohmic losses since the path through the grid network is shorted. So the averaging procedure is not a mathematical trick but implements fundamental physical information about

lateral balancing currents. The original version of the Eqs. (5a) and (5b) calculates all lateral balancing currents. But typically one is interested in lateral balancing currents which significantly influence the voltage distribution across the solar cell. In order to do so it is best to average (smooth) already the original $j_{01}(x, y)$ maps to cancel out all lateral balancing currents on short length scale (e.g. from a good grain into the grain boundaries). This additional averaging has slight but significant influence on the relative variation of average diode currents, e.g. $(j_{01,s} - j_{01})/j_{01}$. The correct/best length scales for lateral smoothing depend on the grid design (emitter resistance, grid finger distance, etc.). The dependence on these length scales of average values across the solar cell is not strong, but in some areas with clustered defect structures significant effects of the averaging length scales become visible, as will be discussed in the experimental part.

For analyzing lateral balancing currents other than under open circuit as discussed up to now the full model including the non-linear diodes of the equivalent circuit in Fig. 1 is solved, especially to calculate changes in I_{lat} . All non-standard information of the parameter in this equivalent circuit are known (have already been extracted from the open-circuit data). The lateral voltage distribution across the solar cell is calculated in linear order in the series resistance using the Taylor expansion

$$U(x, y, I_{ext}, I_{lat}) \approx U(x, y, 0, I_{lat,0}) + R_{s}(x, y)I_{ext} + R_{lat}(x, y)(I_{lat} - I_{lat,0})$$
(7)

This dependence on external current I_{ext} and lateral balancing current I_{lat} will be used to correct open circuit luminescence measurements and series resistance maps extracted from luminescence measurements as well. The quality of the correction will be discussed as a first hint for the correctness of the modelling presented in this paper.

3 EXPERIMENTAL RESULTS AND DISCUSSION

The experimental data shown here represent typical examples out of a large set of luminescence measurements on several solar cells with different defect structure/distribution across the solar cell, each performed at 20 different injection levels for 10 different external currents I_{ext} . The injections have been controlled better than 1% and the temperature for the whole set of measurements has been controlled better than (25 ± 0.1) C°. These precise control showed up to be necessary in order to get reliable/reproducible map data. Part of the explanation for the necessity for this precise control becomes clear from the discussion of the effects of lateral balancing currents as presented here.

Fig. 2 shows several maps for a quite homogeneous solar cell made from high performance mc-Si. Fig. 2a) is the open circuit luminescence map, Fig. 2b) the corresponding $j_{01}(x, y)$ map calculated in Fuyuki approximation mainly using the data of Fig. 2a). The series resistance map is shown in Fig. 2e). Marked in several maps are broken grid fingers. Obviously these broken grid fingers are visible in the open-circuit data as well, being a strong hint that significant lateral balancing currents exist under open circuit condition.



Fig. 2: Results for a quite homogeneous (high performance) mc-Si solar cell. a) Open-circuit luminescence map; b) $j_{01}(x, y)$ map extracted from map a); c) $j_{01}(x, y)$ map corrected for lateral balancing current effects using map d); d) series resistance map of lateral balancing currents calculated as explained in the text; e) standard series resistance map using data with $I_{\text{ext}} = 5.5 \text{ A}$; f) series resistance map of e) corrected for lateral balancing current effects.

A closer look reveals that, seemingly, far away from the main bus bars the grains are often better. These features show again a clear correlation to regions with large series resistances. Fig. 2d) is the series resistance map related to lateral balancing currents calculated as explained in the theory section, combining the information of Fig. 2a) and Fig. 2e). Fig. 2d) has been used to correct Fig. 2c) for lateral balancing current effects. An astonishingly homogeneous map is found, showing nearly no artifacts of the series resistance map. Please note that no free (adaptable) parameter was used to find this result.

This is a clear hint that open-circuit data even for quite homogeneous solar cells can strongly be affected, if not dominated by series resistance effects. Fig. 2f) has been added for completeness and for comparison with the other solar cell results. It shows our series resistance data, corrected for possible lateral balancing currents. No significant improvement/change is visible.

Fig. 3 shows the corresponding results for a mc-Si solar cell with many regions of clustered defects and Fig. 4 for a solar cell where the edge effects of the "red zone" induced by the crucible is clearly visible on two edges. For both cells larger lateral series resistance effects can be expected. In Fig. 3a) and 3b) similar effects are visible as already discussed for Fig. 2, i.e. broken grid fingers are visible in the open-circuit data. Such broken grid fingers are not clearly visible in Fig. 4a) and 4b). The reason for this is *not* that the lateral balancing currents are smaller than in Fig. 3, actually this solar cell has the largest lateral balancing currents.



Fig. 3: Results for mc-Si solar cell with a many locally clustered defects. a) Open-circuit luminescence map; b) $j_{01}(x, y)$ map extracted from map a); c) $j_{01}(x, y)$ map corrected for lateral balancing current effects using map d); d) series resistance map of lateral balancing currents calculated as explained in the text; e) standard series resistance map using data with $I_{ext} = 5.5$ A; f) series resistance map of e) corrected for lateral balancing current effects.

The reason for this is the large homogeneous area with good diffusion length (the source for lateral balancing currents) and the quite small homogeneous area with bad diffusion length (the drain for lateral balancing currents). This distribution has two consequences:

- According to Eq. (5a) the ohmic losses due to lateral balancing currents in the good region are quite small.
- According to Eq. (5b) the ohmic losses due to lateral balancing currents in the bad region are large. This large resistance limits the global lateral balancing

current, since $\left(\frac{R_s}{1-\alpha_d} + \frac{R_s}{\alpha_d}\right)$ is the full resistance the lateral balancing current has to pass.

Just comparing the lateral variation of the $j_{01}(x, y)$ maps in Fig. 3) and 4) much larger lateral balancing current I_{lat} in Fig. 4 would have been expected. As can be seen in Fig. 5 the lateral balancing current I_{lat} of the cell in Fig. 4 (curve C1) is only somewhat larger than that of the cell in Fig. 3 (curve B1).



Fig. 4: Results for mc-Si solar cell with "red zone" around two edges. a) Open-circuit luminescence map; b) $j_{01}(x, y)$ map extracted from map a); c) $j_{01}(x, y)$ map corrected for lateral balancing current effects using map d); d) series resistance map of lateral balancing currents calculated as explained in the text; e) standard series resistance map using data with $I_{ext} = 5.5$ A; f) series resistance map of e) corrected for lateral balancing current effects.



Fig. 5: Dependence of lateral balancing currents I_{lat} on external current I_{ext} and injection. A: for solar cell in Fig. 2; B: for solar cell in Fig. 3; C: for solar cell in Fig. 4. Index 1 represents the data for the injection levels as shown in the figures. Index 2 represents corresponding data measured at lower (by a factor 4.5) injection strength.

In detail Fig. 5 presents the lateral balancing currents I_{lat} as a function of I_{ext} for the three solar cells at the injection level as used in the above figures and for a significantly (a factor 4.5) smaller average injection level. As expected the solar cell shown in Fig. 2 (curve A) has the smallest lateral balancing currents.

The value of the lateral balancing current strongly depends for all solar cells on the injection level. It has a much smaller dependency on the external current I_{ext} . This is the reason why the open-circuit data are more affected by lateral balancing currents than the series resistance maps. The curve C1 shows the steepest slope in Fig. 5. Only for this solar cell the correction of the series resistance map, Fig. 4e), shows a slight change/improvement, cf. Fig. 4f): The asymmetry in series resistance between the left and right hand side of the solar cell becomes smaller. The significant dependence of the lateral balancing current I_{lat} on I_{ext} and on the injection level of the solar cell visible in Fig. 5 and thus the implicit dependence of the lateral potential distribution according to Eq. (7) can easily explain why only the accurate control of the injection level allows for a correct calculation of series resistance data as used in our analysis. In combination with the explicit injection level dependence of series resistance data this can explain why defect characterization methods (relying on the combination of luminescence maps measured at different injection levels) often fail.

The power losses related to lateral balancing currents can easily be calculated using the equation

$$P_{\text{lat}} = \left(\frac{R_s}{1 - \alpha_d} + \frac{R_s}{\alpha_d}\right) I_{\text{lat}}^2$$

We will not discuss details for the individual solar cells but just give a simple approach for the upper limit. Worst case is that source and drain for lateral balancing currents occupy roughly the same area on the solar cell, i.e. 50%, leading to $P_{\text{lat}} \approx 4 R_s I_{\text{lat}}^2$. Compared to the ohmic power losses induced by external current flow $P_s = R_s I_{\text{ext}}^2$ one gets

$$\frac{P_{\text{lat}}}{P_s} \approx 4 \left(\frac{I_{\text{lat}}}{I_{\text{ext}}}\right)^2 \approx 1\%$$

where the number of 1% has been calculated for the solar cell and the measurement condition of Fig. 3. So the additional power losses are not that relevant; lateral balancing current just make the interpretation of map data for inhomogeneous solar cells much more difficult.

4 SUMMARY AND CONCLUSIONS

Similar to the case of code breaking a very robust statistical approach has been presented which just relies on histogram data of $j_{01}(x, y)$ maps and allows to quantify most of the relevant information about lateral balancing currents. For decoding text the real art starts after transferring back the statistical information into the text: several gaps have to be filled and some minor errors for characters not following the expected statistics have to be identified. The same is true for our approach when translating back the histogram information into local lateral balancing current maps. There is still space for improvement. But already the actual approach (using mainly lateral averaging) allows to correct luminescence maps for solar cells with various defect structures significantly and average information about lateral balancing current effects on solar cells is already reliable.

REFERENCES

- [1] J.-M. Wagner, J. Carstensen, A. Berhane, A. Schütt, and H. Föll, "Series resistance analysis with the shaded luminescence technique", in Proc. 26th European Photovoltaic Solar Energy Conference, 2BV.2.51, Hamburg (2011).
- [2] J.-M. Wagner, M. Hoppe, A. Schütt, J. Carstensen, and H. Föll, "Injection-level dependent series resistance:

Comparison of CELLO and photoluminescence-based measurements", Energy Procedia **38**, 199 (2013).

- [3] J. Carstensen, J.-M. Wagner, A. Schütt, and H. Föll, "Relation Between Local and Global I-V Characteristics: Restrictions for and by Series Resistance Averaging", in Proc. 29th European Photovoltaic Solar Energy Conference, 2BV.8.21, Amsterdam (2014).
- [4] J. Carstensen, J.-M. Wagner, A. Schütt, A. Krudopp, and H. Föll, "Ohmic loss analysis for lateral balancing currents by CELLO and photoluminescence measurements", in Proc. 29th European Photovoltaic Solar Energy Conference, 2BO.2.6, Amsterdam (2014).
- [5] S.L. Singh, The Code Book: The Science of Secrecy from Ancient Egypt to Quantum Cryptography, Doubleday, New York (1999).