# RELATION BETWEEN LOCAL AND GLOBAL *I–U* CHARACTERISTICS: RESTRICTIONS FOR AND BY SERIES RESISTANCE AVERAGING

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ABSTRACT: Several approaches have been published for analyzing local I-U curve parameter in order to estimate the contribution of each point of a solar cell to the global I-U curve. Nearly all of them use a one- or two-diode model for the description of the local as well as the global I-U curve. In this paper it will be shown that due to the nonlinear diode characteristics and the distributed series resistance network some fundamental restrictions exist for averaging local parameters; thus in general the translation of local I-U curve parameters into averaged global I-Ucurve parameters does not work. One fundamental condition will be proposed which ensures that nearly all expectations with respect to the applicability of the standard equivalent circuit for describing the global I-U curve of solar cells and the relation to local parameters hold.

Keywords: series resistance, equivalent circuit, local diode parameters, averaging procedures, I-U characteristic

## 1 INTRODUCTION

The standard equivalent circuit, as shown in Fig. 1, implicitly states that current source, ideal diode, and ohmic resistor (series resistance) have fixed parameters which do not depend on the other components. For example, the properties of the ideal diode do not change when varying the series resistance; just the voltage applied to the diode differs from the external voltage by  $R_s \times I_{ext}$ . We will show that this independence holds for (most) distributed 2D networks of local diodes and local series resistances (cf. Fig. 2) in linear order in  $R_s$  and only in linear order in  $R_s$ , i.e. it needs a "good" grid to apply the standard equivalent circuit.

Behind this seemingly trivial aspect lies a second seemingly trivial aspect: It is a standard implicit expectation that local solar cell parameters can be measured, mapped, and the averages of the maps equal the global values. However, for the series resistance this is not trivial at all, as can be seen by the following discussion: In the one-diode model, the global I-U characteristic can be written as

$$I_{\text{ext}} = I_{01} \left( e^{\frac{q(U_{\text{ext}} - R_s I_{\text{ext}})}{kT}} - 1 \right) - I_{\text{ph}}$$

$$= I_{\text{D}} (U_{\text{ext}} - R_s I_{\text{ext}}) - I_{\text{ph}}.$$
(1)

For each position m on the solar cell, one has basically the same equation:

$$I_{\rm m} = I_{01,\rm m} \left( e^{\frac{q(U_{\rm m})}{kT}} - 1 \right) - I_{\rm ph,m} = I_{\rm Dm}(U_{\rm m}) - I_{\rm ph,m},$$
(2)

where  $U_{\rm m}$  is the local voltage, which incorporates the series resistance. There is still an open discussion how to implement local series resistances, either together with the total current,

$$U_{\rm m} = U_{\rm ext} - R_{\rm s.m} I_{\rm ext}, \tag{3a}$$

or with the local current,

$$U_{\rm m} = U_{\rm ext} - R_{\rm s,m} I_{\rm m}.$$
 (3b)

Either way, the standard implicit assumption is that the global I-U characteristic can be written as



**Figure 1:** Standard (simplified) equivalent circuit of a solar cell, consisting of photocurrent source, ideal diode, and series resistance.



**Figure 2:** Standard (simplified) model for local solar cells connected in parallel by a distributed series resistance network.

$$I_{\text{ext}} = \left\langle \sum_{m} I_{m} \right\rangle$$

$$= \left\langle \sum_{m} I_{01,m} \right\rangle \left( e^{\frac{q(U_{\text{ext}} - R_{s} I_{\text{ext}})}{kT}} - 1 \right) - \left\langle \sum_{m} I_{\text{ph},m} \right\rangle,$$
(4)

i.e. all global currents (densities) of Eq. (1) have been replaced by the sums of the local currents (densities), which is by no doubt allowed since currents add up, at least at short-circuit condition, for which Eq. (4) must hold as well. However, combining Eqs. (2) and (4) leads to the contradiction that

$$\left\langle \sum_{m} I_{01,m} e^{\frac{q(-R_{s,m}I)}{kT}} \right\rangle \neq \left\langle \sum_{m} I_{01,m} \right\rangle e^{\frac{q(-R_{s,m}I)}{kT}}$$
(5)

[for either version of the local voltage, Eq. (3), therefore the current *I* appears on the left-hand side without index], since the nonlinearity of exponential function and (real) distributed series resistance network do not fulfill this expectation. So the question arises, why Eqs. (1) and (2) do work at all? In other words: Why does the equivalent circuit, Fig. 1, work at all as replacement for the distributed solar cell network, Fig. 2?

A similar question was raised last year by Micard and Hahn [1] who scrutinized the implicit expectation that all ohmic losses within a solar cell can be calculated just from the overall external current  $I_{ext}$  and the global series resistance  $R_s$  as  $P_{ext} = R_s I_{ext}^2$ . Using a numerical simulation they showed that the sum of all local Joule losses really equals  $P_{ext}$ . We will show that for real solar cells for which standard equivalent circuits as shown in Fig. 1 are a useful representation, ohmic losses can be summarized as discussed above, but that it is not true in general for a distributed network schematically shown in Fig. 2, i.e. the question raised by Micard and Hahn is a fundamental one for the understanding of 2D solar cells.

## 2 SERIES RESISTANCE ANALYSIS

The ideal perfect grid (ignoring all series resistances), allowing for unrestricted lateral balancing currents and leading to perfect equipotential across the solar cell, fulfills all common implicit expectations (that local solar cell parameters can be measured and mapped and that averages of local parameter maps represent the corresponding global values): both, photocurrents and diode currents add up and can therefore be averaged, and all local measurements can be straightforwardly interpreted since the voltage is the same everywhere. Therefore, against standard interpretation, these perfectly parallel connected local diodes can be assumed independent diodes.

We will show that taking into account distributed series resistances up to linear order for a 2D network, all implicit expectations are still fulfilled, i.e. mapping and averaging of local series resistance parameters leads to consistent results for the global cell parameter. This is not trivial, since this is not true for 1D networks! So, the equivalent circuit in Fig. 1 needs a 2D resistance network, i.e. it is not valid for a 1D description of a grid on a 2D area of a solar cell.

The modelling starts by assuming a constant emitter sheet resistance  $\rho$  and a constant area-related diode conductance  $K := A^{-1} dI_D/dU$  (A: area of the solar cell), but the 2D result will show an intrinsic robustness which makes it insensitive to all "random" local variations of resistances as well as of diode properties. So for each working point along the *I*–*U* curve a different value for *K* must be taken in the following. For this linearized description, the standard equivalent circuit model, Fig. 1, is replaced by that of Fig. 3 with a global diode resistance  $R_D$  which is related to *K* by

$$KA = \frac{1}{R_{\rm D}}.$$
(6)

The relation between the lumped series resistance  $R_s$  and the sheet resistance  $\rho$  will be derived below. The linearized version of the local distributed network in Fig. 2 is shown in Fig. 4.



**Figure 3:** Linearized equivalent circuit diagram with photocurrent source (red), linearized diode resistance  $R_D$  (green), and series resistance  $R_s$  (blue).



Figure 4: Linearized version of Fig. 2, described by a constant emitter sheet resistance  $\rho$  and a constant diode conductance *K*.



Figure 5: Simple 2D solar cell model with rotational symmetry, described by a constant emitter sheet resistance  $\rho$  and a constant diode conductance *K*. The boundary condition of Eq. (11) is illustrated.

The basic differential equation for the simple 2D model with rotational symmetry, Fig. 5 (originally introduced and discussed in [2]), is derived from current conservation for the lateral emitter sheet current  $I_{\text{lat}}$ , given by the integrated diode current density  $J_{\text{D}} = KU$ ,

$$I_{\rm lat}(r) = \int_{0}^{r} KU(r') 2\pi r' dr',$$
(7)

and Ohm's law for the lateral sheet current density at radius *r*,

$$\frac{dU}{dr} = \rho \frac{I_{\text{lat}}}{2\pi r},\tag{8}$$

yielding

$$\frac{dU}{dr} = \frac{\rho K}{r} \int_{0}^{r} U(r') r' dr'.$$
<sup>(9)</sup>

In what follows, we will use the abbreviation

$$a := \frac{\pi r^2}{A} \tag{10}$$

for the relative area of the solar cell inside the radius r. The boundary condition

$$\frac{dU}{dr}(r_{\max}) = \rho \frac{I_{ext}}{2\pi r_{\max}}$$
(11)

expresses that at the edge, the lateral current equals the global external current.

The analytical solution of Eq. (9) for this boundary condition is

$$U(r) = \frac{\rho I_{\text{ext}}}{2\pi r_{\text{max}}\sqrt{\rho K}} \frac{I_0(\sqrt{\rho K}r)}{I_1(\sqrt{\rho K}r_{\text{max}})},$$
(12)

where  $I_0$  and  $I_1$  are modified Bessel functions of corresponding order (asymptotically showing an exponential behavior for large values).

Note that for all passive networks  $I_{ext}$  will show up as a scaling factor in the voltage distribution; this is one reason why in our analysis always Eq. (3a) is used for defining local series resistances and why we believe that Eq. (3b) is fundamentally wrong.

Introducing the relative area a, Eq. (10), as variable, Eq. (12) can be rewritten as

$$R(a) = \frac{U(a)}{I_{\text{ext}}} = \frac{\rho}{2\pi \sqrt{\frac{\rho KA}{\pi}}} \frac{I_0\left(\sqrt{\frac{\rho KA}{\pi}}a\right)}{I_1\left(\sqrt{\frac{\rho KA}{\pi}}\right)},$$
(13)

expressing a "relative-area-related diode resistance", its meaning becoming clear below. Solving the differential equation, Eq. (9), up to linear order in  $\rho$  [i.e., using a linear approximation to  $I_0$  in Eq. (13)] and applying the boundary condition we get

$$R(a) = \frac{1}{KA} \frac{1 + \frac{1}{4} \frac{\rho KA}{\pi} a}{1 + \frac{1}{8} \frac{\rho KA}{\pi}} = R_{\rm D} \frac{1 + \frac{1}{4} \frac{\rho}{\pi R_{\rm D}} a}{1 + \frac{1}{8} \frac{\rho}{\pi R_{\rm D}}},\tag{14}$$

where for the second expression, Eq. (6) was used to introduce  $R_{\rm D}$ . Obviously,

(i) since R(a) depends linearly on a and a varies between 0 and 1, the average of R(a) is attained at a = 0.5, and from Eq. (14) one has that  $\langle R(a) \rangle = R(0.5) = R_{\rm D}$ ;

(ii) since  $U(r_{\text{max}}) = U_{\text{ext}}$ , one has that  $R(r_{\text{max}}) = R_{\text{sc}}$ , the global solar cell resistance.

Furthermore, from the global linearized equivalent circuit, Fig. 3, one has that the global series resistance can be obtained as

$$R_{\rm s} = R_{\rm sc} - R_{\rm D}.\tag{15}$$

This allows to introduce a relative-area-dependent series resistance function by defining [3]

$$R_{\rm s}(a) := R_{\rm max} - R(a). \tag{16}$$

This definition has two important consequences which justify the choice for  $R_s(a)$ :

(i) Due to R(a) being a linear distribution, also  $R_s(a)$  is a linear distribution, which allows a simple arithmetic averaging of  $R_s(a)$ , and

(ii) the arithmetic average of  $R_s(a)$  fulfills exactly the relationship of the linear-response equivalent circuit:

$$\langle R(a) \rangle = \langle R_{\max} - R(a) \rangle = R_{\max} - \langle R(a) \rangle$$
  
=  $R(1) - R(0.5) = R_{sc} - R_{D} = R_{s}.$  (17)

Figures 6 and 7 summarize these results, derived from Eqs. (14) and (16):

 the average of the local series resistances equals the global series resistance;

- the average of the local diode resistances equals global diode resistance;

– the average voltage necessary to drive a global current through the local diodes is independent of  $\rho$ , i.e. averaging local diodes is independent of averaging local series resistances.



**Figure 6:** Illustration of Eq. (14) with R(a) vs *a*. The average of the local diode resistance,  $\langle R_D \rangle$ , is the global diode resistance,  $R_D = R(0.5)$ , and the maximum is the global solar cell resistance,  $R_{sc} = R(1)$ .



**Figure 7:** Illustration of Eq. (14) with R(a) vs *a* indicating the calculation of the global series resistance  $\langle R_s \rangle$  according to Eq. (17).

Now, by defining (cf. [4])

$$R_{\rm s,\infty} \coloneqq \frac{1}{8} \frac{\rho}{\pi} \tag{18}$$

we can rewrite Eq. (14) as

$$R(a) = R_{\rm D} \frac{1 + \frac{2R_{\rm s,\infty}}{R_{\rm D}}a}{1 + \frac{R_{\rm s,\infty}}{R_{\rm D}}},$$
(19)

which for the global  $R_s = R_{sc} - R_D = R(1) - R(0.5)$  yields

$$R_{\rm s} = \frac{R_{\rm s,\infty}}{1 + \frac{R_{\rm s,\infty}}{R_{\rm D}}} = \frac{1}{\frac{1}{R_{\rm s,\infty}} + \frac{1}{R_{\rm D}}},\tag{20}$$

i.e.

$$\frac{1}{R_{\rm s}} = \frac{1}{R_{\rm s,\infty}} + \frac{1}{R_{\rm D}}.$$
 (21)

This is the effect of a distributed network: a part of the lateral current is short-circuited by the local diodes, reducing the ohmic losses through the grid!

So although the series resistance changes along the I-U curve for each injection according to Eq. (21), one parameter  $R_s$  exists representing the average series resistance. Using the linearized version, Eq. (14), it is easy to show that all local Joule losses sum up to  $P_{\text{ext}} = R_s I_{\text{ext}}^2$ . This is exactly what Micard and Hahn [1] stated. It is an important check for the series resistance network of real solar cells since the full solution of Eq. (13) does not fulfill this condition in general.

#### 3 GENERALIZATION OF THE SIMPLE DISTRI-BUTED SERIES RESISTANCE MODEL

Using the variable a instead of r [according to Eq. (10)] in the differential equation Eq. (9) we find

$$\frac{dU}{da} = \frac{\rho KA}{4\pi} \frac{1}{a} \int_{0}^{a} U(a') da'.$$
<sup>(22)</sup>

The simplified solution of Eq. (14) linear in  $\rho$  is obtained by taking a constant voltage  $U = R_D I_{ext}$  in the integral. This voltage is just the solution for  $\rho = 0$ . Please note that the factor 1/a is a direct consequence of the 2D model and in consequence enforces dU/da to be constant. The physical reason for this is the increased number of paths for points far away from the contacts, basically compensating the increases ohmic losses along one path. This is the same explanation as for  $\Omega$ /square being the specific sheet resistance.

Essentially a part of Eq. (22) just describes averages over local diode currents, i.e.

$$\langle I_{\rm D} \rangle = KA \frac{1}{a} \int_{0}^{a} U(a') da' = \frac{1}{R_{\rm D}} \frac{1}{a} \int_{0}^{a} U(a') da'.$$
 (23)

So in linear order in  $\rho$  the solution and all consequences described above will not differ if e.g. local diode properties vary randomly from point to point (white unbalanced noise) or if local series resistance and local diode properties are spatially uncorrelated, i.e.

$$\left\langle \left( I_{\rm D}(x,y) - \left\langle I_{\rm D}\right\rangle \right) \left( R_{\rm s}(x,y) - \left\langle R_{\rm s}\right\rangle \right) \right\rangle = 0.$$
(24)

Some essential differences exist for the 1D case. The factor 1/a is missing in Eq. (22) and Eq. (23). Thus dU/da scales with a, i.e. the voltage U(a) linear in  $\rho$ scales with  $a^2$ . So while in the 1D case dU/da scales with the integral of local diode currents, in the 2D case dU/dascales with the average of local diode currents. This induces a significant dependence of the position of local recombination-active diodes to ohmic losses in the 1D case which does not exist in the 2D case. Additionally, unambiguous averaging of diode properties is not possible because their relevance for the global solar cell properties drastically depends on the position relative to the grid; e.g. if a grid finger is broken in the 1D case, all diodes beyond that point do not contribute to the global solar cell properties. In the 2D case a broken grid finger just leads to an increased current flow along ways bypassing the broken grid finger. In summary the factor 1/a in front of the integrals in Eq. (22) and Eq. (23) reflects the intrinsic robustness of a 2D network for bypassing defects; current flow across a 2D network is a strongly nonlocal phenomenon and this is necessary to apply reasonably the standard equivalent circuit shown in Fig. 1 to standard solar cells.

Eq. (23) and (24) can explain why local diode variation has a much less severe influence on local series resistances than often expected, which probably was one reason to apply Eq. (3b) and the model of "independent diodes" (in a star-connected circuit) as e.g. described in [5 - 7].

Further reasons for insignificant variation of local diode slopes are lateral balancing currents over short distances: As long as the approximation of ohmic losses up to linear order in  $\rho$  is valid even for the *whole* solar cell, ohmic losses for lateral balancing current over short distances are negligible. The real challenge are lateral balancing currents over large distances across a solar cell, which are beyond the scope of this work.

Of course, main bus bars, as schematically shown in Fig. 8, effectively reduce the length of the path for lateral current flow relative to the area of a solar cell. As implied by the circles, main bus bars lead to a parallel arrangement of (identical) smaller solar cells, roughly N cells with an area A/N each. Taking into account Eqs. (18)–(21), the consequence for the voltage distribution can be summarized as

$$\widetilde{R}(a) = \frac{R(a)}{N} = R_{\rm D} \frac{1 + 2R_{\rm s,\infty} \frac{a}{NR_{\rm D}}}{1 + R_{\rm s,\infty} \frac{1}{NR_{\rm D}}},$$
(25)

i.e.

$$\frac{1}{R_{\rm s}} = \frac{1}{R_{\rm s,\infty}} + \frac{1}{NR_{\rm D}}.$$
 (26)



**Figure 8:** Two different paths for a current generated on the left flowing to the sink on the right via a direct connection (dashed line) or via the grid (arrow-grid-arrow). The presence of the grid and main bus bar structure virtually separates the solar cell into N = 4 different parts (indicated by the 4 circles).

The main bus bars (+ grid fingers + bulk resistance) induce pure (nondistributed) ohmic losses, so the complete average series resistance of the global solar cell can be written as

$$R_{s} = \frac{1}{\frac{1}{R_{s,\infty}} + \frac{B}{R_{D}}} + R_{s,\text{ nondistributed}}$$
(27)

leading to the equivalent circuit shown in Fig. 9.



**Figure 9:** Equivalent circuit according to Eq. (22). The green resistance is the diode resistance  $R_D$  divided by *B*, while the blue resistances represent the series resistance ( $R_{s,\infty}$  left,  $R_{s,nondistributed}$  right).

This equivalent circuit in combination with the simultaneous averaging procedure for local series and diode resistances as described above leads to consistent and completely quantitative results for mono- and many multicrystalline solar cells using the CELLO [2, 8] as well as photoluminescence [3, 9, 10] techniques.

# 4 SUMMARY

The standard interpretation of a global I-U curve needs an independent averaging of local solar cell parameters, e.g. local series and diode resistances. This is in general only possible in linear order in the sheet resistance  $\rho$  and for a 2D resistance network.

The average local diode resistances are not influenced by the sheet resistance  $\rho$ . But the average local series resistance depends on the average diode resistance, i.e.  $R_s$  changes along the *I*-*U* curve.

The reason WHY an analysis (only) in linear order in the sheet resistance  $\rho$  is enough for analyzing solar cells is NOT a physical one! It is the large effort of the process engineers to design and produce emitters and grids good enough for an efficient current collection from all parts of a solar cell.

In consequence this is as well the reason for the applicability of the standard equivalent circuit.

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