MODELLING OF THE DISTRIBUTED SERIAL GRID RESISTANCE:
VERIFICATION BY CELLO MEASUREMENTS
AND GENERALIZATION TO OTHER RESISTANCE MAPPING TOOLS

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ABSTRACT: We will give a definition for "economically reasonable solar cells" and discuss some very general properties of such cells. The only reason, why probably most of the commercially available solar cells and nearly all solar cells which we have investigated with CELLO fulfill these conditions is that everybody wants to sell or buy only such solar cells. The two basic statements for economically reasonable solar cells are that (independent of the serial resistance network) all local diode resistances add up in parallel to the inverse global diode resistance and that (independent of the local diode resistances) all local serial resistances add up to the global serial resistance. As a result of the modelling of the serial resistance network both statements can be summarized in one statement which is not so intuitive as the above ones but which opens the path to a quite universal procedure to calculate quantitative maps from the measurement result of a large number of measurement tools: the voltage distribution across a solar cell is a linear function of the solar cell area.
Keywords: characterization, modeling.

1 INTRODUCTION

Almost all solar cells with a reasonable efficiency which we have investigated with CELLO (solar CELl LOcal characterization) [1 - 3] in the last years showed a linear voltage distribution as a function of area (details see below). For such solar cells with a linear voltage distribution a quite simple procedure has been developed [3] to get a quantitative map of the local serial resistances, so it is really helpful to find a straight line in the corresponding plot. But the really astonishing thing is, that for all solar cells this straight line has been found, so the question arises, under which general condition a linear voltage distribution across the area of a solar cell can be expected.

It is a common knowledge and a common expectation that the power of a solar cell scales with the area of the solar cell. This is e.g. essential for the definition of the efficiency of a solar cell. For a perfect grid (R_{ser} = 0) this is obviously true. All diode resistances $R_{Di} \propto 1 / A_i$ from small areas A_i sum up in parallel with the same quantity to the full inverse diode resistance

$$\frac{1}{R_D} = \sum_i \frac{1}{R_{Di}} \propto \sum_i A_i = A_{SC} \cdot \qquad (1)$$

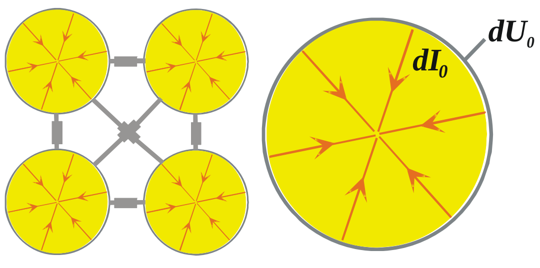
Eq. (1) explains why the specific diode resistance has a dimension of Ω cm^2.

A corresponding scaling law is expected obviously for all solar cells, i.e. the grid should be good enough to allow for an efficient current collection from all (diode) parts of a solar cell. If such a grid is not possible or too expensive it is better to produce smaller solar cells and connect these cells externally in a module. Otherwise large parts of a solar cell - especially those far away from the main bus bars - could be cut off without significant power losses. We will call solar cells which fulfill such a basic requirement as a scaling law between the power and the area "economically reasonable". "Economically reasonable" solar cells can have quite bad efficiencies, as we will see later. Why a scaling law with area is not a strong restriction, as one might expect in a first glance will become clear later on as well. In what follows some quite fundamental properties of economically reasonable solar cells will be discussed.

2 GENERAL SOLAR CELL MODEL

The final result of a general economical solar cell could probably be derived in several ways. Here we will summarize results which have already been published [3]. **Fig. 1** shows a schematic drawing of a solar cell configured as a network of resistances and (small) circular solar cells. The circular shape is not essential for the description, it just allows for a simple analytical solution of the current-voltage distribution of this device. This circular solar cell has a metallization on the back side, a pn-junction with a diode resistance $R_D = 1 / (K\,A)$ (A: area, K scaling factor) and a metal contact around the periphery to the emitter at the front side with sheet resistance ρ. Driving a current dI_0 through the cell the voltage U as a function of radius r is given by

$$dU(r) = \frac{dI_0 \rho}{2\pi \sqrt{\rho K}\, r_{\max}} \frac{I_0(\sqrt{\rho K}\, r)}{I_1(\sqrt{\rho K}\, r_{\max})}. \qquad (2)$$



a) b)

Figure 1: a) Schematic representation of a solar cell as a network of resistances and circular solar cells. b) illustration of one circular solar cell element.

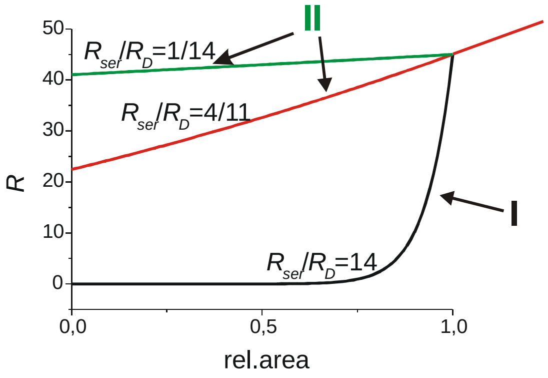
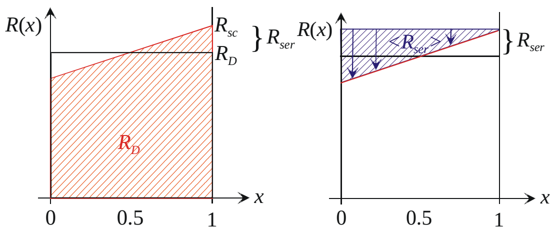


Figure 2: Examples for the voltage curves of Eq. (2). Case I: exponential behavior for large ratio of serial resistance / diode resistance. Case II: linear behavior for small ratio of serial resistance / diode resistance.

Here I_0 and I_1 are modified Bessel functions. Solutions of Eq. (2) are shown in **Fig. 2** for several ratios of serial resistances / diode resistances and a new variable $x := (\pi r^2)/A$. Case I shows the result which is generally expected: The voltage changes exponentially. Obviously the current driven into this solar cell only reaches a small part of the cell. If illuminating such a cell only from that part photo current would be collected efficiently. This result does in principle need no calculation to be found. If the voltage changes exponentially across the area of a solar cell, than the power of such a solar cell can not scale with it's area. According to our definition such a solar cell is not economically reasonable. Only the case II where a linear voltage distribution is found can be economically reasonable. The generalization to the complete network in **Fig. 1 a)** is quite clear. If for certain areas of a solar cell not all parts contribute to the power, than this is true for the whole solar cell as well. If for all parts of a solar cell the voltage distribution shows a linear relation to the area, than this is true for a network of such elements and pure resistors, at least if the solar cell is isotropic, i.e. all circular elements are nearly identical (resistors just lead to an offset in the voltage but do not change slopes with respect to the area). So for economically reasonable solar cells the Tailor expansion of Eq. (2) up to linear order in the area must hold. Using the definition $R(x) := dU(x)/dI_0$, we finally get

$$R(x) = \frac{1}{KA}\,\frac{1+\frac{1}{4}\frac{\rho K A x}{\pi}}{1+\frac{1}{8}\frac{\rho K A}{\pi}} = R_D\,\frac{1+\frac{1}{4}\frac{\rho x}{\pi R_D}}{1+\frac{1}{8}\frac{\rho}{\pi R_D}}\,. \qquad (3)$$



a) b)

Figure 3: Example of Eq. (3) for economically reasonable solar cells which illustrate the averaging rules for a) diode resistances and b) serial resistances.

An example for this equation is shown in **Fig. 3**. Of course the averaging rule for diode resistances can be derived easily analytically but it is illustrative to have a closer look to **Fig. 3a)**. We find the same averaging rule of Eq. (1) although all parts contribute differently to the average of $R_D = 1 / (K\,A)$. This result just reflects charge conservation and would be found for the non economically reasonable solar cells as well.

Before discussing the above result in detail we will derive the averaging law for serial resistances. The resistance which we find at $x = 1$ in **Fig. 3 a)** is that on the grid of this solar cell, i.e. the resistance of the global solar cell R_{sc}. Since we did not include shunts into our consideration according to **Fig. 4** the difference between the global solar cell resistance and the global diode resistance is the global serial resistance R_{ser}. All lateral resistance losses are induced by ohmic losses. **Fig. 3b)** summarizes these results for the serial resistance. Quite obviously, but very astonishingly we find that the average of all local serial resistance losses equals the global serial resistance. This is the same result which we would get if just putting N identical resistors $R_{ser,i} = R_{ser} / N$ in series

$$R_{ser} = \sum_{i=1}^{n} R_{ser,i}\,. \qquad (4)$$

For economically reasonable solar cells there is a large symmetry between the averaging law for local diodes in Eq. (1) and local serial resistances in Eq. (4). Local diodes add up in parallel to the global diode resistance with the same law one finds for the ideal case ($R_{ser} = 0$). Local serial resistances add up to the global serial resistance with the same law one finds for the ideal case ($R_D = \infty$). But in contrast to the ideal cases different parts of the solar cell contribute differently to the global values: Due to the ohmic losses the local diodes are on different potentials. Due to the local diodes different currents flow through the local serial resistances.

We have defined economically reasonable solar cells by having a scaling law for the solar cell power with the area of the solar cell. To fulfill this condition it is not necessary to have a perfect grid, it is sufficient to produce a grid for which ohmic losses do not increase stronger than linearly with the area of the solar cell. This is reflected in Eq. (3) and **Fig. 3**. To get an economically reasonable solar cell according to Eq. (2) roughly $R_{ser} < R_D$ must hold. So as a rule of thumb: When short circuit currents can be extracted from all parts of the solar cell, the cell is most probably "economically reasonable".

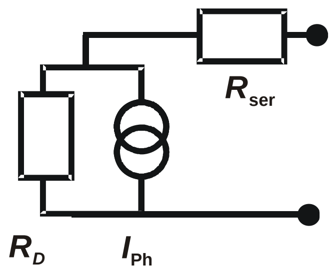


Figure 4: Very simple equivalent circuit of a standard solar cell.

There is a last criterion which allows to predict if a solar cell is definitely economically reasonable. The relation between a linear voltage distribution in **Fig. 3** and the standard equivalent circuit in **Fig. 4** has already been mentioned before. It is quite astonishing that a distributed network which mixes local diodes and local resistances can be replaced by the equivalent circuit in **Fig. 4** of two independent resistors in series and with the simple averaging laws of Eq. (1) and Eq. (4). While R_D will change as a function of applied voltage, the serial resistance in the standard model is expected to be constant. Defining $R_{ser,\infty} := \rho / (8\,\pi)$ and using $R_{ser} = R(1)$ in Eq. (3) we find

$$\frac{1}{R_{ser}} = \frac{1}{R_{ser,\infty}} + \frac{1}{R_D} \quad . \qquad (5)$$

So according to Eq. (5) as long as the global diode resistance R_D is much higher than the global serial resistance R_{ser}, the serial resistance will not change significantly even if R_D changes, i.e. the applied voltage to the solar cell changes. But as soon as $R_D \approx 4\, R_{ser}$ the serial resistance will significantly become smaller. The reason for this is the current which is short circuited by the diode and therefore does not flow through the (local) series resistance network. As a consequence the standard equivalent circuit does not hold any more although the solar cell is still economically reasonable. But of course, as long as the standard equivalent circuit can be used for modeling a solar cell iv-curve, it will be most probably economically reasonable.

So summing up, being economically reasonable is not a strong restriction to a solar cell. It just reflects properties, which nearly everybody would expect for all solar cells. The definition still has one important consequence: a linear voltage distribution across the solar cell area. Although for nearly all possible networks of local diodes and local series resistances the condition of being economically reasonable will not be fulfilled, for nearly all commercially available solar cells it will be fulfilled, just because otherwise they would have a very poor efficiency. Everybody wants to sell good solar cells. The reason why nearly all solar cells show a linear voltage distribution is neither a physical nor a technical one, it is an economical one and therefore we have chosen the name accordingly.

3 QUANTITATIVE MAPS FOR LOCAL SERIAL RESISTANCES

For any method which allows to measure maps which are sensitive to the local voltage distribution it is a good idea to check for the voltage distribution as a function of the area. As discussed above, there is a good chance to find a straight line. The only question is how to calculate the distribution function from a map in a simple way. Having generated a resistance map, just the histogram $H(R)$ is needed.

$$N(R) := \int_0^R H(R)\,dR \quad x(R) := \frac{N(R)}{N(\infty)} \quad \Rightarrow \; R(x) \qquad (6)$$

According to the steps in Eq. (6) by integrating (summing) up the histogram a "density" function $N(R)$ is generated. A real density function $x(R)$ is calculated by dividing $N(R)$ by the full area respectively the full number of pixels in the image. Inverting this function the final result $R(x)$ is calculated which corresponds to Eq. (3). If a straight line is found, the solar cell is economically reasonable and the Eq. (1) to (4) can be applied.

Often, as it is e.g. the case for the CELLO maps, the voltage sensitive maps contain some not very precisely known scaling factors and offsets. But if e.g. from a fit to the global iv-curve the global diode resistance R_D and the global serial resistance R_{ser} are known, Eq. (3) respectively the results of **Fig. 3** can be used to generate quantitative maps for local serial resistances and/or local diodes.

Fig. 5 summarizes CELLO results for the analysis of local serial resistances.

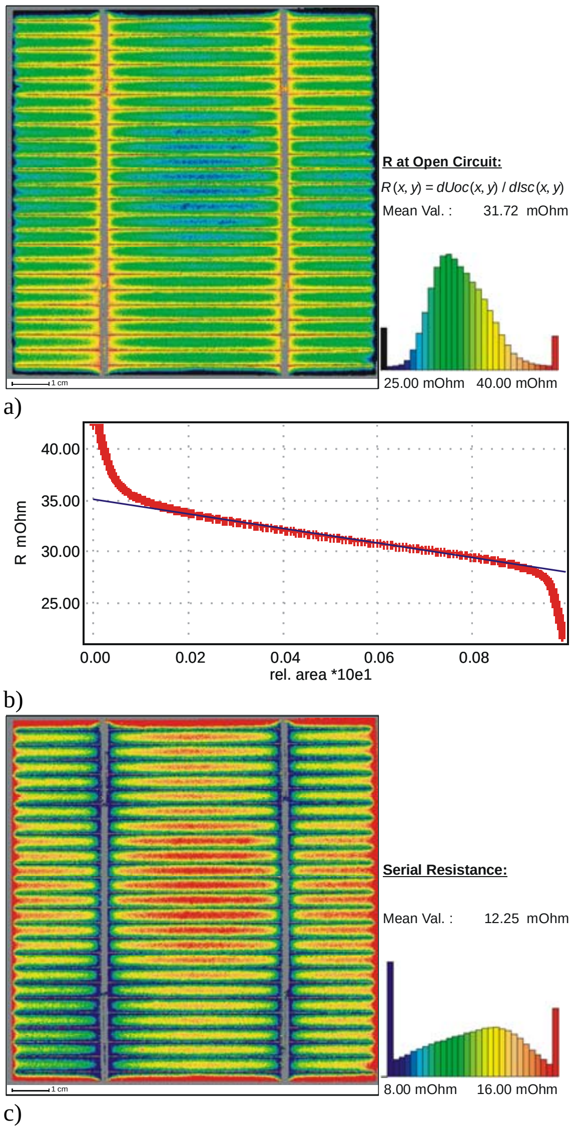


Figure 5: Typical CELLO example for serial resistance analysis. a) Resistance map calculated from open circuit map and short circuit map; b) voltage distribution function calculated from Eq. (6); c) serial resistance map using Eq. (3) for calibration to the global serial resistance R_{ser}.

There is one decisive difference between the two straight lines in **Fig. 3a)** and **Fig. 5b)**. In **Fig. 5b)** the resistances are arranged just according to their values *R*. In **Fig. 3a)** the resistances are arranged according to their distance from the metal contact. Since $R(x)$ in Eq. (3) is a monotone/linear function this has the same effect as arranging resistances like in **Fig. 5b)**. Any imperfection in the solar cell will somehow modify the resistance map, leading to a deviation from the perfect straight line for very small and very large x values. This does not mean, that some parts of the solar cell are not economically reasonable, it just means that the proposed algorithm does not put all elements to their right position (according to the distance from the main bus bars). Taking into account this errors, it is even more astonishing that more than 80 % of the solar cell area reflect one straight line.

4 GENERALIZATION TO OTHER METHODS

Several measurement tools like photoluminescence, electroluminescence, or IR thermography can produce maps which are sensitive to the local voltage. For such measurement tools which generated data that is directly proportional to the local voltage the proposed method of Eq. (6) can of course be directly applied. In some cases the measurement tools produce maps $G(x,y)$ which are only indirect functions of the local voltage $F(U(x,y))$. If the transfer function F is known the proposed procedure can be applied to $F^{-1}(G(x,y))$.

5 SUMMARY AND OUTLOOK

We introduced a quite weak definition for an economically reasonable solar cell. It defines mainly a requirement for the grid: The grid design should allow for a power extraction from the solar cell which scales with the size of the cell. This does not need for a perfect solar cell. As has been shown, very bad solar cells can fulfill this definition as well. But economically reasonable solar cells must show a linear voltage distribution as a function of the area. This is indeed a new information about solar cells which is necessarily fulfilled for nearly all commercially available solar cells. In this paper it has been used to discuss a procedure for calculating serial resistance maps which can be applied to a variety of measurement tools which generate maps sensitive to the local voltage distribution.

6 ACKNOWLEDGEMENTS

This work has been supported by the German network project "Netz Diagnistik" and by DFG grant FO 258/11-1.

7 REFERENCES

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