Solution to Exercise 2.1-2

Derive numbers for v_0, v_D, τ, and l

First Task: Derive a number for v_0 (at room temperature). We have:

$$v_0 = \left(\frac{3kT}{m}\right)^{1/2} = \left(\frac{3 \cdot 8.6 \cdot 10^{-5} \cdot 300}{9.1 \cdot 10^{-31}} \; \frac{\text{eV} \cdot \text{K}}{\text{K} \cdot \text{kg}}\right)^{1/2} = 2.92 \cdot 10^{14} \cdot \left(\frac{\text{eV}}{\text{kg}}\right)^{1/2}$$

- The dimension "square root of **eV/kg**" does not look so good - for a velocity we would like to have **m/s**. In looking at the energies we equated kinetic energy with the classical dimension **kg · m²/s² = J** with thermal energy kT expressed in **eV**. So let's convert **eV** to **J** (use the link) and see if that solves the problem. We have $1\ \text{eV} = 1.6 \cdot 10^{-19}\ \text{J} = 1.6 \cdot 10^{-19}\ \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$, which gives us

$$v_0 = 2.92 \cdot 10^{14} \cdot \left(\frac{1.6 \cdot 10^{-19}\ \text{kg} \cdot \text{m}^2}{\text{kg} \cdot \text{s}^2}\right)^{1/2} = 1.17 \cdot 10^5\ \text{m/s} = 4.21 \cdot 10^5\ \text{km/h}$$

Possibly a bit surprising - those electrons are no sluggards but move around rather fast. Anyway, we have shown that a value of ≈ 10^4 **m/s** as postulated in the backbone is really OK.

- Of course, for $T \to 0$, we would have $v_0 \to 0$ – which should worry us a bit???? If instead of room temperature (**T = 300 K**) we would go to **1200 K**, let's say, we would just double the average speed of the electrons.

Second Task: Derive a number for τ. We have:

$$\tau = \frac{\sigma \cdot m}{n \cdot e^2}$$

First we need some number for the concentration of free electrons per m^3. For that we complete the table given, noting that for the number of atoms per m^3 (i.e, the atomic density) we have to divide the density by the atomic weight.

Atom	Density [kg · m^{-3}]	Atomic weight [1.66 · 10^{-27} kg]	Conductivity σ [10^7 Ω^{-1} · m^{-1}]	Atomic dens. [10^{28} m^{-3}]
Na	970	23	2.4	2.54
Cu	8,920	64	5.9	8.40
Au	19,300	197	4.5	5.90

- So let's take $5 \cdot 10^{28}\ \text{m}^{-3}$ as a good order of magnitude guess for the number of atoms in a m^3, and for a first estimate some average value $\sigma = 5 \cdot 10^7\ \Omega^{-1}\ \text{m}^{-1}$. We obtain

$$\tau = \frac{5 \cdot 10^7 \cdot 9.1 \cdot 10^{-31}}{5 \cdot 10^{28} \cdot (1.6 \cdot 10^{-19})^2} \; \frac{\text{kg} \cdot \text{m}^3}{\Omega \cdot \text{m} \cdot \text{A}^2 \cdot \text{s}^2} = 3.55 \cdot 10^{-14} \; \frac{\text{kg} \cdot \text{m}^2}{\text{V} \cdot \text{A} \cdot \text{s}^2}$$

Well, somehow the whole thing would look much better with the unit **s**. So let's see if we can remedy the situation.

- Easy: volt times ampere equals *watt*, which is power, i.e. energy per time, with the unit $\text{J} \cdot \text{s}^{-1} = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$. Insertion yields

$$\tau = 3.55 \cdot 10^{-14} \frac{kg \cdot m^2 \cdot s^3}{kg \cdot m^2 \cdot s^2} = 3.55 \cdot 10^{-14}\ s = 36\ fs$$

- The backbone thus is right again. The scattering time is in the order of femtoseconds, which is a short time indeed. Since all variables enter the equation linearly, looking at somewhat other carrier densities (e.g. more than **1** electron per atom) or conductivities does not really change the general picture very much.

- Third Task: Derive a number for v_D. We have (for a field strength E **= 100 V/m = 1 V/cm**):

$$|v_D| = \frac{E \cdot e \cdot \tau}{m} = \frac{100 \cdot 1.6 \cdot 10^{-19} \cdot 3.55 \cdot 10^{-14}}{9.1 \cdot 10^{-31}} \frac{V \cdot C \cdot s}{m \cdot kg} = 6.24 \cdot 10^{-1} \frac{V \cdot A \cdot s^2}{m \cdot kg}$$

$$= 6.24 \cdot 10^{-1} \frac{kg \cdot m^2 \cdot s^2}{m \cdot kg \cdot s^3} = 6.24 \cdot 10^{-1}\ m/s = 624\ mm/s$$

- This is somewhat larger than the value given in the backbone text.
  - However - a field strength of **1 V/cm** applied to a *metal* is huge! Think about the current density j you would get if you apply **1 V** to a piece of metal **1 cm** thick.
  - It is actually $j = \sigma \cdot E = 5 \cdot 10^7\ \Omega^{-1}\ m^{-1} \cdot 100\ V/m = 5 \cdot 10^9\ A/m^2 = 5 \cdot 10^5\ A/cm^2$!
  - For a more "reasonable" current density of $10^3\ A/cm^2$ we have to reduce E hundredfold and then end up with $|v_D|$ = **6.24 mm/s** – and that is slow indeed!

- Fourth Task: Derive a number for l. We have:

$$l = 2 \cdot v_0 \cdot \tau = 2 \cdot 1.17 \cdot 10^5 \cdot 3.55 \cdot 10^{-14}\ m = 8.31 \cdot 10^{-9}\ m = 8.31\ nm$$

  - Right again! If we add the comparatively miniscule v_D, nothing would change.
  - Note, however, that the last equation does NOT mean that decreasing the temperature would lower l to eventually zero! Why? Because τ isn't a constant but scales with the conductivity – look at the starting equation of the second task! And since the conductivity increases at lower temperatures, so does the mean free path.