Solution to Exercise 2.1-1

Derive and discuss numbers for μ and v_D

First Task: Derive numbers for the mobility **μ**.

- First we need typical conductivities and electron densities in *metals*, which we can take from the table in the link.
- At the same time we expand the table a bit

Material	ρ [Ω cm]	σ [Ω^{-1} cm^{-1}]	**Density *d* [10^3 kg m^{-3}]**	**Atomic weight *w* [1u = 1.66 · 10^{-27} kg]**	***n* = *d*/*w* [m^{-3}]**
Silver (Ag)	$1.6 \cdot 10^{-6}$	$6.2 \cdot 10^5$	10.49	107.9	$5.85 \cdot 10^{28}$
Copper (Cu)	$1.7 \cdot 10^{-6}$	$5.9 \cdot 10^5$	8.92	63.5	$8.46 \cdot 10^{28}$
Lead (Pb)	$21 \cdot 10^{-6}$	$4.8 \cdot 10^4$	11.34	207.2	$3.3 \cdot 10^{28}$

For the mobility **μ** we have the equation

$$\mu = \frac{\sigma}{q \cdot n}$$

With *q* = elementary charge **= 1.60 10^{-19} C** and the density of electrons = density of atoms ***n*** calculated above, we obtain, for example for μ_{Ag}

$$\mu_{Ag} = \frac{6.2 \cdot 10^5}{1.6 \cdot 10^{-19} \cdot 5.85 \cdot 10^{28}} \; \frac{m^3}{C \cdot \Omega \cdot cm} = 66.2 \; \frac{cm^2}{C \cdot \Omega}$$

The unit is a bit strange, but rembering that **1 C = 1 A · 1 s** and **1 Ω = 1 V / 1 A**, we obtain

$$\mu_{Ag} = 66.2 \; \frac{cm^2}{Vs}$$

$$\mu_{Cu} = 43.6 \; \frac{cm^2}{Vs}$$

$$\mu_{Pb} = 9.1 \; \frac{cm^2}{Vs}$$

Second Task: Derive numbers for the drift velocity v_D by considering a reasonable field strength.

- The mobility **μ** was defined as

$$\mu = \frac{v_D}{E}$$

or

$$v_D = \mu \cdot E$$

So what is a reasonable field strength in a metal?

- Easy. Consider a cube with side length ***l* = 1 cm**. Then, the relevant area is ***F* = 1 cm²**, and its resistance ***R*** is given by

$$R = \frac{\rho \cdot l}{F} = \rho\ [\Omega]$$

- A **Cu** or **Ag** cube thus would have a resistance of about $\mathbf{1.5 \cdot 10^{-6}\ \Omega}$. Applying a voltage of **1 V**, or equivalently a field strength of **1 V/cm** thus produces a current of ***I* = *U*/*R*** ≈ **650 000 A** or a current density of ***j* = 650 000 A/cm²**
- That seems to be an awfully large current. Yes, but it is the kind of current *density* encountered in integrated circuits! Think about it!
- Nevertheless, the wires in your house carry at most about **30 A** (above that the fuse blows) with a cross section of about **1 mm²**; so a reasonable current density is **3000 A/cm²**, which we will get for about $\mathbf{U = 1.5 \cdot 10^{-6}\ \Omega \cdot 3000\ A = 4.5\ mV}$.
- For a rough estimate we then take a field strength of **5 mV/cm** and a mobility of **50 cm²/Vs** and obtain

$$v_D = 50 \cdot 5\ \frac{\text{mV} \cdot \text{cm}^2}{\text{cm} \cdot \text{V} \cdot \text{s}} = 0.25\ \frac{\text{cm}}{\text{s}} = 2.5\ \frac{\text{mm}}{\text{s}}$$

That should come as some surprise! The electrons only have to move v e r y s l o w l y *on average* in the current direction (or rather, due to sign conventions, against it).

- Is that true, or did we make a mistake?
- It *is* true! However, it does *not* mean that electrons will not run around like crazy inside the crystal, at very high speeds. It only means that their *net* movement in current (anti-)direction is very slow.
- Think of an single fly in a fly swarm (even better, read the module that discusses this analogy in detail). The flies are flying around at high speed like crazy – but the fly swarm is not going anywhere as long as it stays in place. There is then no drift velocity and no net fly current!