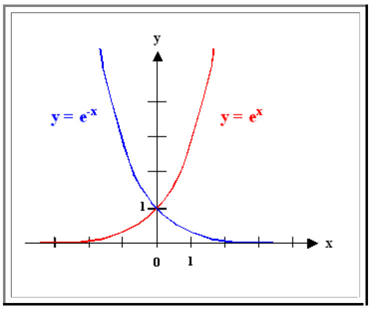
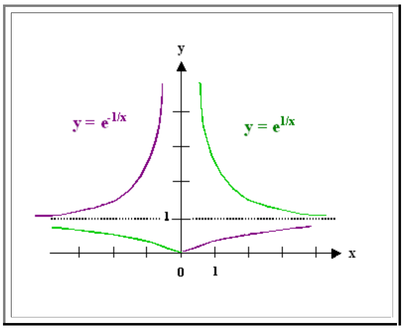
Working with Exponentials and Logarithms

Basics

- First, the graphical representation of the most important exponential curves (see math script as well)

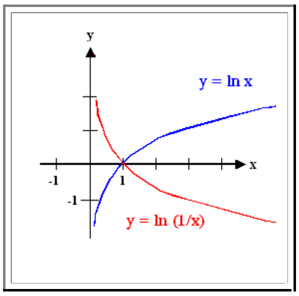




- The typical curves everybody should know. The blue curve in the first quadrant (positive ***x*** values) corresponds to the *energy dependence* of the ubiquitous **Boltzmann factor exp – (*E*/ k*T*)**

- Slightly more tricky. Note that the purple branch in the **1.** quadrant corresponds to the *temperature dependence* of the ubiquitous Boltzmann factor **exp – (*E* /k*T*)**

- The inverted functions, e.g. **y = ln x** are easily pictured, too; below the **y = ln x** and the **y = ln (1/x)** functions are shown.



- The graphs, in case you forgot, also illustrate some basic algebraic relations, e.g.
  - $e^{-x} = 1/e^{x}$
  - $\ln x = -\ln (1/x)$

- The essential identities are

$$e^{x} = \frac{1}{e^{-x}} \qquad (e^{x})^{y} = e^{x \cdot y}$$

$$e^{x} \cdot e^{y} = e^{x+y} \qquad (e^{x})^{1/y} = e^{x/y}$$

$$\frac{e^{x}}{e^{y}} = e^{x-y} \qquad e^{\ln x} = x$$

$$\ln (x \cdot y) = \ln x + \ln y$$

$$\ln \frac{x}{y} = \ln x - \ln y$$

$$\ln x^{y} = y \cdot \ln x$$

- Here some approximations.

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ..$$

$$\ln (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ..$$

$$\ln (1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ..\right)$$

- While many equations contain exponential terms of some variable which "disappear" if you substitute the **ln** of the variable (as in the **Arrhenius plot**), we mostly prefer the **lg** of some observable quantity to the **ln**. As an example, plotting the vacancy concentration c_V in an Arrhenius plot would be straight forward with the **ln**:
  - On order to get a straight slope we have to switch to new variables according to

$$c_V = A \cdot \exp - \frac{H^F}{kT}$$

$$\ln c_V = \ln A - \frac{H^F}{kT} \cdot \frac{1}{T}$$

$$y = \ln A - \frac{1}{T} \cdot x$$

for

$$y = \ln c_V \quad x = \frac{1}{T}$$

- For ***y*** and *x* we get straight line with slope $-H^F/k$ and intercept = ***A***

What do we have to do if we want to plot **lg** **(c_V)** instead of **ln (c_V)**?

- We have to multiply everything with **lg e = 0,4342..**, because

$$\lg x = (\lg e) \cdot (\ln x)$$ [1)]

- We obtain

$$\lg (c_V) = 0{,}434 \cdot \ln A - 0{,}434 \cdot \frac{H_V}{k} \cdot \frac{1}{T}$$

1) If this puzzles you, consider

- We postulate the equality

$$\lg x = M \cdot \ln x$$

- and want to find a value for **M**. Raising everything to the power of **10** gives

$$10^{\lg x} = x = 10^{M \cdot \ln x}$$

- This equation can only be fulfilled if

$$10^M = e$$

- because then we have $e^{\ln x} = x$ as required, giving **M = lg e**