

# Units and Constants

## General Remarks

This is the no-nonsense module with the hard facts about units, constants and transformations from one system of units into another one (after this paragraph, that is).

No explanations, historical roots, really outdated or unusual units are given - for the [fun part](#) use the link.

## Basics

First, the *basics*:

- In physics we always have two things: a *physical quantity* - e.g. the speed of something, or the strain of something under load - *and* some units to measure the quantity in question.
- The *physical quantity* is what it is - it does not depend on how *you* express it in numbers. Somebody on some other planet will for sure do it differently from you and me.
- The *number* you will give to the physical quantity is strictly a function of the units *you* chose. You might use **m/s**, oder **lightyears/s**, or **wersts/year** - that will just change the *number* for the speed of the moving object a lot, but not the speed itself. Trivial, but often forgotten.

To make life easier for everybody (at least for scientists), the choice of units was taken away from you and me, and everybody is now required to *strictly adhere* to the **international standard system**, abbreviated in any language as **SI** units.

Well, by now you, and I, and most others scientists, do comply with the **SI** system ([which was not always the case](#)) - but the public at large does not give shit; especially in the **USA**. Tell the gas station attendant any number you like in **pascal** or **bar** for the tire pressure, and he (or she) will just look at you as if you escaped from the lunatic asylum. Its **psi** or bust! And on occasion, even engineers or scientists do *not* use **SI** units - with [disastrous consequences](#) if you have tough luck.

The question now is: how many **basic units** do we need, so we can express *everything else* in these units? And which ones do we take?

- This is one of the deeper questions of humankind. Physicists claim that we just need *one more truly* basic constant of nature - and we do not need units *at all* anymore. Velocities, for instance, can always be given using the absolutely constant speed of light (in vacuum) as the unit; your typical car speed than would be something like **0.000,001**.
- But redundancy tends to make life easier (just look at your typical Sheik and his harem), and the **SI** system gives us **7 basic units** which are independent of each other.

## Basic Units

| Quantity                  | Unit name | Symbol |
|---------------------------|-----------|--------|
| Length                    | meter     | m      |
| Mass                      | kilogram  | kg     |
| Time                      | second    | s      |
| Electrical current        | ampere    | A      |
| Thermodynamic temperature | kelvin    | K      |
| Amount of substance       | mol       | mol    |
| Luminous intensity        | candela   | cd     |

Note that in English only the names of persons (as well as of animals and fictitious characters) are written with the first letter capitalized. Therefore, all units must be written with **small letters** only. (The same holds for the chemical elements, by the way: small letters only!)

From this basic units all other **SI** units can be derived. Below are tables with the more important secondary units.

- First, we look at some secondary units just invoking basic units *and* a length. While we often do use special symbols for these quantities (e.g.  $\rho$  for density), these symbols are not really necessary and thus were not pronounced immutable and sacred as, e.g., the "m" for meter or the "s" for second.

| Quantity                   | Unit name                | Symbol                              |
|----------------------------|--------------------------|-------------------------------------|
| Area                       | square meter             | m <sup>2</sup>                      |
| Volume                     | cubic meter              | m <sup>3</sup>                      |
| Velocity                   | meter per second         | m/s; ms <sup>-1</sup>               |
| Acceleration               | meter per square second  | m/s <sup>2</sup> ; ms <sup>-2</sup> |
| Wave number                | reciprocal meter         | m <sup>-1</sup>                     |
| Density                    | kilogram per cubic meter | kg/m <sup>3</sup>                   |
| Specific volume            | cubic meter per kilogram | m <sup>3</sup> /kg                  |
| Electrical current density | ampere per square meter  | A/m <sup>2</sup>                    |
| Magnetic field strength    | ampere per meter         | A/m                                 |
| Substance concentration    | mol per cubic meter      | mol/m <sup>3</sup>                  |
| Luminance                  | candela per square meter | cd/m <sup>2</sup>                   |

Now some more involved units - including important quantities like *energy*, *voltage*, and *magnetic* things.

- They are more involved, because we usually do *not* express them in **SI** basic units - which is perfectly possible - but in *secondary* units. We will also find one case where there is *no unit* - it just cancels out.
- These units often have their own symbols for reasons that become clear if you look at the **SI** units, and these symbols should not be used for something else

| Quantity                                   | Unit name | Symbol | Conversion         |   |
|--|-----------|--------|--------------------|---|
|  |           |        | in secondary units | in basic units  |
| Plane angle                                | radian    | rad    |                    | m / m = 1   |
| Frequency                                  | hertz     | Hz     |                    | s <sup>-1</sup>   |
| Force                                      | newton    | N      |                    | m · kg · s <sup>-2</sup>  |
| Pressure, stress                           | pascal    | Pa     | N/m <sup>2</sup>   | m <sup>-1</sup> · kg · s <sup>-2</sup>                            |
| Energy, work, quantity of heat             | joule     | J      | N·m                | m <sup>2</sup> · kg · s <sup>-2</sup>                             |
| <b>Power</b> , energy flux                 | watt      | W      | J/s                | m <sup>2</sup> · kg · s <sup>-3</sup>                             |
| Quantity of electricity<br>Electric charge | coulomb   | C      |                    | A·s   |
| Electric potential, voltage                | volt      | V      | W/A                | m <sup>2</sup> ·kg·s <sup>-3</sup> ·A <sup>-1</sup>               |
| Capacitance                                | farad     | F      | C/V                | m <sup>-2</sup> ·kg <sup>-1</sup> ·s <sup>4</sup> ·A <sup>2</sup> |
| Electric resistance                        | ohm       | Ω      | V/A                | m <sup>2</sup> ·kg·s <sup>-3</sup> ·A <sup>-2</sup>               |
| Conductance                                | siemens   | S      | A/V                | m <sup>-2</sup> ·kg <sup>-1</sup> ·s <sup>3</sup> ·A <sup>2</sup> |
| Magnetic flux                              | weber     | Wb     | V·s                | m <sup>2</sup> ·kg·s <sup>-2</sup> ·A <sup>-1</sup>               |
| Magnetic flux density                      | tesla     | T      | Wb/m <sup>2</sup>  | kg·s <sup>-2</sup> ·A <sup>-1</sup>                               |

|                     |                               |    |      |  |
|---------------------|-------------------------------|----|------|--|
| Inductance          | henry                         | H  | Wb/A | $\text{m}^2 \cdot \text{kg} \cdot \text{s}^{-2} \cdot \text{A}^{-2}$ |
| Celsius temperature | degree celsius ("centigrade") | °C |      | K  |
| Radioactivity       | becquerel                     | Bq |      | 1/s  |

● Again: By using small letters it is clear that here it's all about the unit names; capitalizing the first letter would refer to the person after which this unit was named.

▸ Mercifully, the members of the "Comité international des poids et mesures" are human (up to a point, at least). In consequence they did *not* outlaw all older units in one fell stroke, but sorted them into *three* groups:

● "Old" units which may be used together with **SI** units *without restrictions*.

● Old units which may be used *for some time* in parallel to **SI** units.

● Old units which are definitely out and *must not be used at all* any more.

▸ Some of the units in the second category are regional and you probably have never heard of them. We will not include them here. The number of outlawed units is legion, we just include the still tempting ones.

▸ Here is the first category: Some of the non-**SI** units *you still may use without restrictions* :

| Unit name              | Symbol | Conversion  |
|------------------------|--------|---|
| minute                 | min    | 1 min = 60 s  |
| hour                   | h      | 1h = 60 min = 3600 s                                  |
| day                    | d      | 1 d = 24 hr = 86400 s                                 |
| angle degree           | °      | $1^\circ = (\pi/180) \text{ rad}$                     |
| angle minute           | '      | $1' = (1/60)^\circ$                                   |
| angle second           | "      | $1'' = (1/60)' = (1/3600)^\circ$                      |
| liter                  | l, L   | $1 \text{ l} = 1 \text{ dm}^3 = 10^{-3} \text{ m}^3$  |
| ton                    | t      | $1 \text{ t} = 10^3 \text{ kg}$                       |
| electron volt          | eV     | $1 \text{ eV} = 1.602,176,6 \cdot 10^{-19} \text{ J}$ |
| atomic mass unit (amu) | u      | $1 \text{ u} = 1.660,539,1 \cdot 10^{-27} \text{ kg}$ |

● What a relief!

▸ Now to the *old units you may use for some more time* to come in parallel to the **SI** units:

| Unit name           | Symbol | Conversion   |
|---------------------|--------|--|
| angstrom / ångström | Å      | $1 \text{ Å} = 0.1 \text{ nm}$   |
| ar                  | a      | $1 \text{ a} = 100 \text{ m}^2$  |
| hectar              | ha     | $1 \text{ ha} = 100 \text{ a}$   |
| bar                 | bar    | $1 \text{ bar} = 0.1 \text{ MPa}$  |
| barn                | b      | $1 \text{ b} = 100 \text{ fm}^2 = 10^{-28} \text{ m}^2$                            |
| curie               | Ci     | $1 \text{ Ci} = 3.7 \cdot 10^{10} \text{ Bq}$                                      |
| roentgen            | R      | $1 \text{ R} = 2.58 \cdot 10^{-4} \text{ C/kg} = 2.58 \cdot 10^{-4} \text{ As/kg}$ |

● Note that the letter Å is not pronounced as the a in "far", instead, it sounds like the o in "of" (cf. [en.wiktionary.org/wiki/%C3%85ngstr%C3%B6m](http://en.wiktionary.org/wiki/%C3%85ngstr%C3%B6m)). Germans seem to think that it has to be pronounced as a mixture of German o and a (i.e., as an a-ish variant of o), but that's wrong!

Now to the *units you must not use anymore!*. We might put them into two groups:

1. The forerunners of the **SI** units, the **cgs units**; i.e. the units based on the *centimeter*, the *gram* and the *second*.
2. The simple old fashioned no-no's.

While it may appear that the **cgs** system is practically the same as the **SI** system, *this is not so!*

- Of course, the *cm*, *g*, and *s* are essentially the same basic units as in the **SI** system, the abbreviation "**cgs**", however, does not tell you anything about the other necessary basic units in this system - and *that* is where the problems come in!
- In fact, there were *several* **cgs** systems - the *electrostatic*, the *electromagnetic*, and the *Gauss cgs* system!
- We will not unravel all the intricacies for **cgs** systems and the conversion to **SI** units here - this is done in its [own module](#) - but just give some of the more common units and their conversion.

| Unit name | Symbol | Conversion  |
|-----------|--------|---|
| erg       | erg    | 1 erg = $10^{-7}$ J                                       |
| dyne      | dyn    | 1 dyn = $10^{-5}$ N                                       |
| poise     | P      | 1 P = 1 dyn·s/cm <sup>2</sup> = 0.1 Pa·s                  |
| gauss     | Gs, G  | 1 G corresponds to $10^{-4}$ T                            |
| maxwell   | Mx     | 1 Mx (= 1 G·cm <sup>2</sup> ) corresponds to $10^{-8}$ Wb |
| oersted   | Oe     | 1 Oe (= 1 dyn/Mx) corresponds to $(1000/4\pi)$ A/m        |

- The "corresponds to" instead of simply "=" is an indication that while the three quantities in question do have **SI** units that correspond to magnetic flux density, magnetic field strength, and magnetic flux, they are *not* exactly the same thing.

Finally, some still fondly remembered old units *you simply do not use anymore* :

| Unit name                                       | Symbol | Conversion   |
|---|--------|--|
| torr  | Torr   | 1 Torr = $(101,325/760)$ Pa<br>$\approx 133.32$ Pa |
| physical atmosphere                             | atm    | 1 atm = 101,325 Pa                                 |
| kilopond  | kp     | 1 kp = 9.806,65 N                                  |
| calorie   | cal    | 1 cal = 4.184 J                                    |
| micron<br>(micro <i>meter</i> is what you use!) | $\mu$  | 1 $\mu$ = 1 $\mu$ m                                |

## Fundamental Constants

Fundamental constants are some numbers with units that cannot (yet) be calculated from some physical theory, but must be measured.

- This may have three possible reasons:

1. There is presently no theory, and there *never* will be a theory, that allows us to calculate fundamental constants. They have the value they have because an act [of one or more gods and/or goddesses](#), or they are purely random (i.e we just happen to live in an universe, where the value is what we measure. In some other universe, or some other corner of our universe, it will be arbitrarily different).
2. There is presently no theory, but some day there will be one. Some fundamental constants will then be calculated and then are no longer fundamental.
3. There already is a theory, or at least a general theoretical framework; we just are not yet smart enough to see the obvious or to do the numerics. Masses of elementary particles, e.g., might be "fundamental constants" that fall into this category.

- Hot-shot physicists have some ideas, which constant might fall into which category. Speculations along this line are a lot of fun - but of no consequence so far. So *I* will not dwell on this. (Of course, *you* may check for yourself which one of the three possibilities you are going to embrace and thus get some idea of what kind of person you are).

- Fundamental physical theories usually introduce one new fundamental constant. Mechanics (including gravitation) needs the gravity constant **G**, quantum theory has Planck's constant **h**, statistical thermodynamics introduces Boltzmann's constant **k**, the special theory of relativity (or Maxwell's theory of electromagnetism which is really part of the relativity theory) needs the speed of light **c**.

- New theories sometimes "explain" old constants of nature because they can calculate them, or replace them by something more fundamental. Boltzmann's constant **k**, for example, is more fundamental than the "fundamental" gas constant **R**, because it relates its number to a fundamental unit of matter (**1 particle**) and not to an arbitrary one like **1 mol**.

- How many truly fundamental constants are there? Why do they have the values they have? (Just slight deviations in the values of some constants would make carbon based life impossible; this is where the so-called "[anthropic principle](#)" comes in). Will we eventually be able, with a "Theory of Everything" (**TOE**) to calculate all natural constants?

- Nobody knows. We run against the deepest physical questions at this point.

- So let's just look at what we have. Since it is customary to list as natural constants some quantities that are actually computable from others, we include some of these "constants" here, too (together with the conversion formula).

| Symbol and formula                      | Numerical value  | Magnitude and unit   | Remarks   |
|---|------------------|--|---|
| <b>Speed of light in vacuum</b>         |                  |  |   |
| $c_0, c$                                | 2.997,924,58     | $10^8 \text{m}\cdot\text{s}^{-1}$                          | Truly fundamental                                     |
| <b>Gravitational constant</b>           |                  |  |   |
| <b>G</b>                                | 6.673            | $10^{-11} \text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$ | Truly fundamental                                     |
| <b>Planck's constant</b>                |                  |  |   |
| <b>h</b>                                | 6.626,068,76     | $10^{-34} \text{J}\cdot\text{s}$                           | Truly fundamental                                     |
|   | 4.135,6          | $10^{-15} \text{eV}\cdot\text{s}$                          |   |
| <b>Elementary charge</b>                |                  |  |   |
| <b>e</b>                                | 1.602,176,462    | $10^{-19} \text{C}$  | Truly fundamental ?<br>Maybe not                      |
| <b>Fine structure constant</b>          |                  |  |   |
| $\alpha = \mu_0 \cdot c \cdot e^2 / 2h$ | 7.297,352,533    | $10^{-3}$  | Unitless, maybe more fundamental than others.         |
| <b>Mass of a electron at rest</b>       |                  |  |   |
| $m_e$                                   | 9.109,381,88     | $10^{-31} \text{kg}$                                       | Not truly fundamental; can be calculated in principle |
|   | 0,510 998 902    | MeV  |   |
| <b>Mass of a proton at rest</b>         |                  |  |   |
| $m_p$                                   | 1.672,621,58     | $10^{-27} \text{kg}$                                       | Not truly fundamental, can be calculated in principle |
|   | 1.007,276,466    | u  |   |
|   | 938.271,998(38)  | MeV  |   |
| <b>Avogadro constant</b>                |                  |  |   |
| $N_A$                                   | 6.022,141,99(47) | $10^{23} \text{mol}^{-1}$                                  | Not truly fundamental any more                        |

| <b>Faraday constant</b>                  |                 |  |  |
|--|-----------------|--|--|
| $F = e \cdot N_A$                        | 96,485.3415(39) | $C \cdot mol^{-1}$                       | Not truly fundamental any more                           |
| <b>Universal gas constant</b>            |                 |  |  |
| R  | 8.314,472(15)   | $J \cdot mol^{-1} \cdot K^{-1}$          | Not truly fundamental any more                           |
| <b>Boltzmann constant</b>                |                 |  |  |
| $k = R/N_A$                              | 1.380,650,3     | $10^{-23} J \cdot K^{-1}$                | Truly fundamental  |
|  | 8.617,269       | $10^{-5} eV \cdot K^{-1}$                |  |
| <b>Magnetic permeability of vacuum</b>   |                 |  |  |
| $\mu_0 = 1/\epsilon_0 c^2$               | 12.566,370,614  | $10^{-7} V \cdot s \cdot A^{-1} m^{-1}$  | Not truly fundamental                                    |
| <b>Electric susceptibility of vacuum</b> |                 |  |  |
| $\epsilon_0 = 1/\mu_0 c^2$               | 8.854,187,817   | $10^{-12} A \cdot s \cdot V^{-1} m^{-1}$ | Not truly fundamental                                    |
| <b>Magnetic flux quant</b>               |                 |  |  |
| $\Phi = h/2e$                            | 2.067,833,636   | $10^{-15} Wb$                            | Smallest possible magnetic flux<br>Not truly fundamental |