2.14.1 Examples: Scalar product

Let

$$\vec{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{R} \qquad \vec{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \in \mathbb{R}$$

be two vectors with real components:

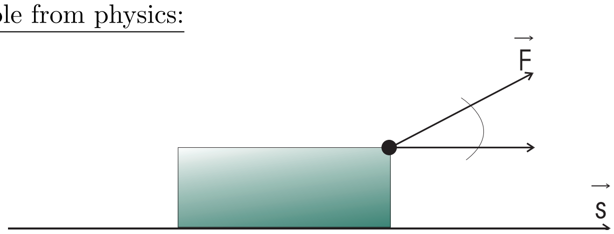
Definition 29 $\vec{a} \cdot \vec{b} = a_1 b_1 + \ldots + a_N b_N = \sum_{j=1}^{k} a_j b_j$ *is called scalar product of* $\vec{a}$ *and* $\vec{b}$
Note: $\vec{a}, \vec{b} \in \mathbb{R}^N, \vec{a} \cdot \vec{b} \in \mathbb{R}$*!!*

Example:

$$\begin{aligned} \vec{a} &= \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 5 \\ 6 \\ 7 \\ 8 \end{pmatrix} \Rightarrow \vec{a} \cdot \vec{b} &&= 5 + 12 + 21 + 32 \\ & &&= 70 \\ \vec{a} &= \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \vec{a} \cdot \vec{b} &&= 1 - 1 + 0 = 0 \end{aligned}$$

Example from physics:

Work:



$$W = \vec{F} \cdot \vec{s} = \left|\vec{F}\right| \, |\vec{s}| \cdot \cos\varphi$$

$$\left.\begin{aligned} \cos\angle\left(\vec{a}, \vec{b}\right) &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot \left|\vec{b}\right|} \\ \vec{a} \cdot \vec{b} &= 0 \quad \text{if} \quad \vec{a} = \vec{0} \quad \text{or} \quad \vec{b} = \vec{0} \quad \text{or} \quad \vec{a} \perp \vec{b} \end{aligned}\right\} \text{in 3D}$$

now definitions for $\vec{a}, \vec{b} \in \mathbb{R}^N$

(i) $\vec{a}, \vec{b} \neq \vec{0}$. If $\vec{a} \cdot \vec{b} = 0$ then $\vec{a}$ is called orthogonal or perpendicular to $\vec{b}$ or $\vec{a} \perp \vec{b}$

(ii) $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \, |\vec{b}|} \leq 1$. We define $\cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \, |\vec{b}|}$ where φ is the angle between the two vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}$

Example:

(i)

$$\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \quad \Rightarrow \quad \cos\varphi = \frac{2 + 2 + 0}{\sqrt{3}\sqrt{8}} = \sqrt{\frac{2}{3}} \rightarrow \varphi = 0.615 = 35.26°$$

(ii)

$$\begin{aligned} \vec{a} &= \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \cos\varphi = \frac{2 + 2 + 0 + 0}{\sqrt{4}\sqrt{8}} = \frac{1}{\sqrt{2}} \Rightarrow \varphi = \frac{\pi}{4} = 45° \\ \vec{a} &= \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \Rightarrow \vec{a} \cdot \vec{b} = 1 - 1 - 1 + 1 = 0 \rightarrow \vec{a} \perp \vec{b} \end{aligned}$$