2.9.1 Examples for the calculation rules for determinants

(a) A change of two columns switches the sign of $\det(\tilde{A})$

example: $\begin{vmatrix} 2 & 3 & 0 \\ 0 & 0 & 1 \\ 4 & 3 & 4 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ 4 & 4 & 3 \end{vmatrix}$

(b) A common (scalar) factor of a column can be taken in front of $\det(\tilde{A})$

example: $\begin{vmatrix} 2 & 3 & 0 \\ 0 & 0 & 1 \\ 4 & 3 & 4 \end{vmatrix} = 3 \begin{vmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 1 & 4 \end{vmatrix} = 6 \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 3 & 4 \end{vmatrix}$

(c) Two determinants which are equal up to one column can be added according to

example: $\begin{vmatrix} 2 & 3 & 0 \\ 0 & 0 & 1 \\ 4 & 3 & 4 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 4 & 3 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 3+1 & 0 \\ 0 & 0+2 & 1 \\ 4 & 3+3 & 4 \end{vmatrix}$

(d) If a multiple of a column is added to another column the $\det(\tilde{A})$ is not changed,

example: $\begin{vmatrix} 2 & 3 & 0 \\ 0 & 0 & 1 \\ 4 & 3 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 3+5\cdot 2 & 0 \\ 0 & 0+5\cdot 0 & 1 \\ 4 & 3+5\cdot 4 & 4 \end{vmatrix}$

(e) $\det(\tilde{A}) = 0$ if column vectors are linearly dependent,

example: $\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$

(f) $\det(\tilde{A}) = \det(\tilde{A}^\top)$, hence <u>all</u> rules are equivalent for lines.
<u>Examples:</u>

$$D = \begin{vmatrix} 2 & 9 & 9 & 4 \\ 2 & -3 & 12 & 8 \\ 4 & 8 & 3 & -5 \\ 1 & 2 & 6 & 4 \end{vmatrix} \underset{=}{\text{II-2I} = \text{II}} \begin{vmatrix} 2 & 5 & 9 & 4 \\ 2 & -7 & 12 & 8 \\ 4 & 0 & 3 & -5 \\ 1 & 0 & 6 & 4 \end{vmatrix} \underset{=}{\text{III}/3} 3\cdot \begin{vmatrix} 2 & 5 & 3 & 4 \\ 2 & -7 & 4 & 8 \\ 4 & 0 & 1 & -5 \\ 1 & 0 & 2 & 4 \end{vmatrix}$$

$$= 3\left\{ -5 \begin{vmatrix} 2 & 4 & 8 \\ 4 & 1 & -5 \\ 1 & 2 & 4 \end{vmatrix} - 7 \begin{vmatrix} 2 & 3 & 4 \\ 4 & 1 & -5 \\ 1 & 2 & 4 \end{vmatrix} \right\} = 0 - 21 \begin{vmatrix} 2 & 3 & 4 \\ 4 & 1 & -5 \\ 1 & 2 & 4 \end{vmatrix}$$

$$\underset{=}{\text{I-III} = \text{I}} -21 \begin{vmatrix} 1 & 1 & 0 \\ 4 & 1 & -5 \\ 1 & 2 & 4 \end{vmatrix} = -21 \left(\begin{vmatrix} 1 & -5 \\ 2 & 4 \end{vmatrix} - \begin{vmatrix} 4 & -5 \\ 1 & 4 \end{vmatrix} \right) = 147$$

(g) other rules:

$$\det(\tilde{I}) = 1$$

$$\det(\tilde{A}\cdot\tilde{B}) = \det(\tilde{A})\det(\tilde{B})$$

$$\text{in general: } \det(\tilde{A}+\tilde{B}) \neq \det(\tilde{A}) + \det(\tilde{B})!$$