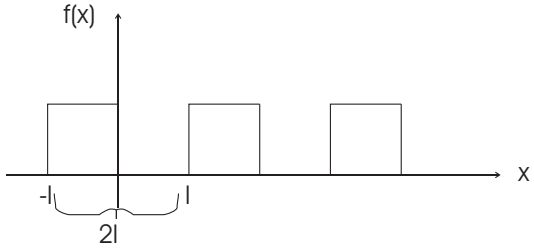
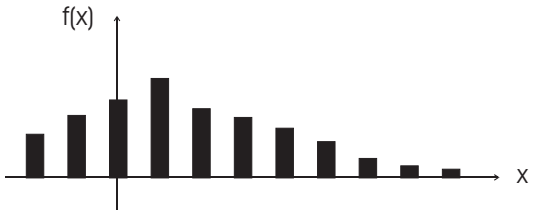
3.12.1 Example: Fourier-Series with larger periodicity length

Periodic function $f(x) = f(x + 2l)$ period 2l



$$f(x) = \sum_{n=-\infty}^{\infty} F_n e^{ip_n x} \qquad p_n = n\frac{\pi}{l} \qquad F_n = \frac{1}{2l}\int_{-l}^{+l} f(x)e^{-p_n x}dx$$



$$\begin{aligned}
p_n &\Rightarrow \text{periodic function} \quad \text{vs.} \quad \text{discrete spectrum} \\
\text{e.g.} \quad f(x) &= \begin{cases} -a & \text{for } -\pi < x < 0 \\ +a & \text{for } 0 < x < \pi \end{cases} \quad l = \pi \\
\rightarrow f(x) &= \frac{4a}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\right) \\
f_n &= \begin{cases} 0 & \text{for } n \text{ even} \\ \frac{4a}{\pi}\frac{1}{n} & \text{for } n \text{ odd} \end{cases}
\end{aligned}$$